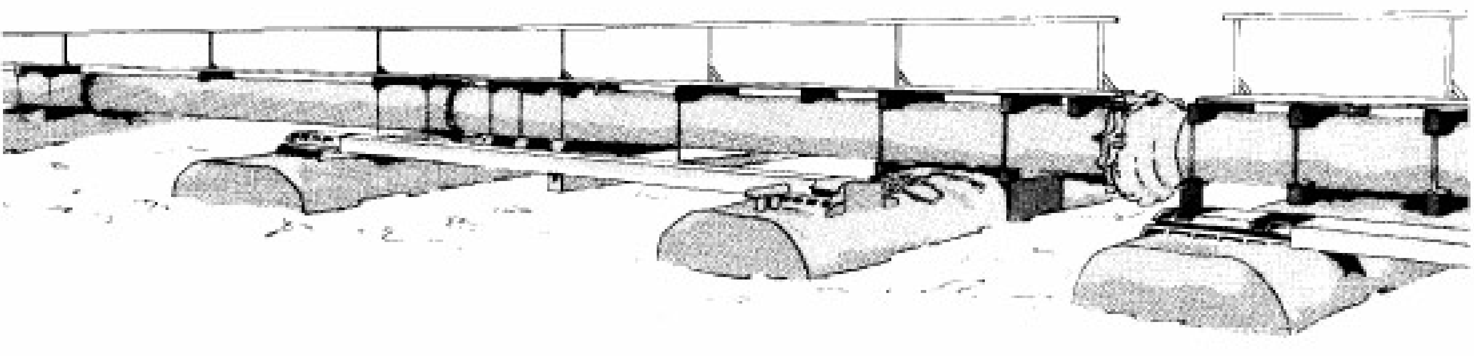
DREDGED MATERIAL RESEARCH PROGRAM

TECHNICAL REPORT D-74-1

INVESTIGATION OF MATHEMATICAL MODELS FOR THE PHYSICAL FATE PREDICTION OF DREDGED MATERIAL

by

B. H. Johnson



March 1974

Sponsored by Office of Dredged Material Research

Conducted by U. S. Army Engineer Waterways Experiment Station
Hydraulics Laboratory
Vicksburg, Mississippi

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IN REPLY REFER TO: WESVS

1 April 1974

SUBJECT: Transmittal of Technical Report D-74-1

TO: All Report Recipients

1. The technical report transmitted herewith represents the results of the first of a planned series of efforts to be accomplished under Task 1B (Fate of Dredged Materials) of the Corps of Engineers' Dredged Material Research Program (DMRP). This task is included under the Aquatic Disposal Project of the DMRP, which is a broad, multifaceted investigation into the environmental impacts and aspects of open water disposal of dredged material.

2. Regardless of the character of the disposal site, an integral part of the problem of assessing the environmental impact of disposal operations is the ability to determine the spatial and temporal distribution of the disposed material. One aspect of this problem, the physical fate or dispersion of the material over relatively short periods of time (2 to 3 days) in the vicinity of the disposal site, appears to be resolvable through the use of mathematical modeling. Although considerable work has been done on the modeling of transport phenomena, particularly those relating to water quality, very little attention has been directed toward the special considerations involved in the modeling of sediment or combinations of particulate matter.

3. The investigation reported herein is an intensive review of the state-of-the-art to determine the availability of a mathematical model suitable or adaptable for use in regard to the disposal of dredged material. The study has revealed that a model developed by R. C. Y. Koh and Y. C. Chang is conceptually well designed and offers the greatest potential for use; however, in its present form, it is applicable only to disposal operations in the ocean environment, and no field and very little laboratory verification data are available.

4. As a result of this investigation, it is concluded that the Koh-Chang model is worthy of further consideration and development and that it should be subjected to field verification. It is also believed that the state-of-the-art is sufficiently advanced to warrant early efforts

1 April 1974

directed at the development of a model specifically adapted to the estuarine environment or at the modification of the existing Koh-Chang model so that it may be applied to disposal operations in an estuary. Steps have already been taken to plan and implement efforts under the DMRP to accomplish both of these objectives.

G. H. HILT
Colonel, Corps of Engineers
Director

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ARMY-MRC VICKSBURG, MISS.

FOREWORD

The study reported herein was conducted at the U. S. Army Engineer Waterways Experiment Station (WES) during the period March 1973-September 1973. It was sponsored by the Office of Dredged Material Research (ODMR) under the civil works research program, "Dredged Material Research Program."

Dr. B. H. Johnson, Mathematical Hydraulics Division, conducted the study and prepared this report under the general supervision of Messrs. H. B. Simmons, Chief of the Hydraulics Laboratory, and M. B. Boyd, Chief of the Mathematical Hydraulics Division. Dr. J. W. Keeley managed the project for ODMR.

Directors of WES during the conduct of this study and the preparation and publication of this report were BG E. D. Peixotto, CE, and COL G. H. Hilt, CE. Technical Director was Mr. F. R. Brown.

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NOTATION

a	Proportionality constant
A	Cross-sectional flow area
c	Instantaneous concentration
$\overline{c}$	Time-averaged concentration
$\tilde{c}$	Depth-averaged concentration
c'	Deviation of the instantaneous concentration from the time-averaged concentration
c''	Deviation of the time-averaged concentration from the cross-sectionally averaged concentration
C	Cross-sectionally averaged concentration
C_b	Average concentration of sediment on a streambed
C_d	Drag coefficient
C_o	Concentration at time $t = 0$ (also, characteristic concentration)
d	Channel depth
D	Molecular diffusion coefficient
e	Vertical diffusion coefficient
e_{cx}, e_{cy}, e_{cz}	Sediment diffusion coefficients in the x, y, and z directions
e_1, e_2, e_3	Turbulent diffusion coefficients in the x, y, and z directions
e_{2_n}	Vertical diffusion coefficient in the case of no density gradient
E	Longitudinal dispersion coefficient
E_x, E_y	Dispersion coefficients
f	Rainfall rate
g	Gravitational acceleration
h	Average channel depth

i	Infiltration rate
j	Particle diameter
k	Coefficient of added mass
K	Horizontal diffusion coefficient in the ocean
ℓ	Scale of turbulent phenomena
L	Distance downstream from the injection of a tracer source
m	Proportionality constant
N	Mean concentration in a generalized Gaussian model
$\tilde{q}_{L_1}, \tilde{q}_{L_2}$	Lateral inflows
Q	Amount of waste released per unit time
r	Hydraulic radius
R	Particle radius
R_e	Reynolds number
R_i	Richardson number
SF	Shape factor
S_y, S_z	Standard deviations in the lateral and vertical directions in a generalized Gaussian model
T	Time interval used in time averaging
$u(t)$	Average velocity in the x direction which is a function only of time t
u, v, w	Instantaneous velocity components
$\overline{u}, \overline{v}, \overline{w}$	Time-averaged velocity components
$\tilde{u}, \tilde{v}, \tilde{w}$	Depth-averaged velocity components
u', v', w'	Deviation of the instantaneous velocity components from the time-averaged velocity components
u'', v'', w''	Deviation of the time-averaged velocity components from the cross-sectionally averaged velocity components
U, V, W	Cross-sectionally averaged velocity components
U^*	Shear velocity
$\overline{U}$	Mean current velocity within the waste field
$\tilde{U}, \tilde{W}$	Horizontal velocities given by a depth-averaged hydrodynamic model
W	Vertical water velocity
W_o	Relative particle speed
W_p	Instantaneous particle velocity

W_s	Particle settling velocity
y_o	Flow depth
Z_o	Amplitude of the vertical water velocity
Z, X	Constants in the modified form of Rubey's equation
β	Undetermined proportionality constant
$\partial/\partial t$	Rate of change with respect to time
$\partial/\partial_x, \partial/\partial_y, \partial/\partial_z$	Rate of change with respect to the spatial coordinates
ε	Rate of energy dissipation
$\bar{\varepsilon}$	Mean value of vertical diffusivity
ε_f	Fluid turbulent mass transfer coefficient
ε_s	Sediment turbulent mass transfer coefficient
$\varepsilon_u, \varepsilon_w$	Velocity distribution functions
$\varepsilon_x, \varepsilon_y$	Turbulent diffusion coefficients
ζ	Distance from the point of maximum surface velocity to the most distant bank
ζ_1	Expression for a variable streambed
ζ_2	Expression for a variable water surface
η	Dimensionless depth
η_1	Expression for a variable left bank of a stream
η_2	Expression for a variable right bank of a stream
λ	Rate of erosion
μ	Dynamic viscosity
ν	Molecular viscosity
ρ	Fluid density
ρ_p	Particle density
σ	Standard deviation
σ_b	Standard deviation at the level of neutral buoyancy
τ	Dimensionless time
ψ	Shape factor
ψ_1	Angle between $\tilde{q}_{L_1}$ and the y axis
ψ_2	Angle between $\tilde{q}_{L_2}$ and the y axis
ω	Angular frequency

CONVERSION FACTORS, BRITISH TO METRIC UNITS OF MEASUREMENT

British units of measurement used in this report can be converted to metric units as follows:

Multiply	By	To Obtain
inches	25.4	millimeters
feet	0.3048	meters
feet per second	0.3048	meters per second
square feet per second	0.092903	square meters per second
pounds	0.45359237	kilograms

SUMMARY

A literature search of technical journals coupled with contacts with other research groups has revealed that very little mathematical modeling of the physical fate of dredged material disposed of in an aquatic environment has been undertaken. The most significant modeling effort that has been found is a mathematical model for prediction of dispersion and settling in barged ocean disposal of wastes developed by R. C. Y. Koh and Y. C. Chang. This model allows for disposal of dredged material by instantaneous bottom dump as well as pumping the material through a pipe under a moving barge. In both disposal operations, the material is traced through three possible phases; namely, convective descent, dynamic collapse, and long-term diffusion. The dynamic collapse is also generalized to account for the possibility that the cloud hits the bottom. The major limitations of the model appear to be:

a. The model was strictly developed to study disposal in an ocean environment.

b. There has been only limited laboratory and no field verification of the model; however, it should be noted that the model is conceptually well designed.

For estuarine and riverine environments, no models capable of tracing dredged material from its initial release into the water column until it is stored on the bottom have been found. However, for the riverine environment, Schroeder and his associates at Oregon State University are currently developing a mathematical model for tracing dredged material released by pipeline discharge. The model is based upon pipeline discharge velocity, ambient fluid velocity, and particle settling velocity. Additional information should be obtained concerning the development and verification of this model to assess its applicability.

As a result of the investigation of identified models and relevant transport studies, the following recommendations are offered:

a. In the ocean environment, sensitivity analyses and field verification of the Koh-Chang model are needed.

b. Model development in the area of predicting the short-term fate of dredged material in the vicinity of the disposal site is needed for the estuarine environment.

c. No model development is recommended for the river disposal problem until further investigation of Schroeder's work is completed.

INVESTIGATION OF MATHEMATICAL MODELS FOR THE PHYSICAL FATE PREDICTION OF DREDGED MATERIAL

PART I: INTRODUCTION

Background

1. Annually, millions of tons of dredged material from streams, estuaries, and coastal waters are disposed of in the aquatic environment. The accurate placement of dredged material is essential in any disposal operation and can be a major problem in aquatic disposal as evidenced by the fact that dredged material has often been found in locations far removed from the designated site. The environment into which the material is disposed, the character of the material, and the method of disposal are the major factors in the determination of the ultimate location and concentration of dredged material.

2. Environmental conditions range from those occurring in the oceans, where turbulent diffusion is the major transport mechanism, to those in estuaries, in which tidal effects and density currents play a role. In addition, there must also be concern with river disposal, where convection is the dominant factor in the transport of mass. The nature of dredged material, quite naturally, is dependent upon the location from which it is obtained. Generally speaking, the materials dredged and disposed of in inland waterways are sand and gravel, whereas in lakes, harbors, and many areas of the coastal zones, the dredged materials often consist of small, light particles such as clays and silts.[1] Dredged material disposed of in the riverine environment is released by means of pumping through pipelines. In practice, the material is quite often sprayed against a large flat plate and allowed to fall to the water surface. Changing the angle of inclination of the plate makes use of the pumped material's momentum to move the end of the pipeline. The disposal of material in the estuarine environment may be accomplished by bottom dump from a hopper dredge or barge or by pipeline discharge.

Disposal of dredged material in the ocean is accomplished only by bottom dump from hopper dredges and barges.

3. A mathematical model of the physical processes determining the ultimate fate of dredged material, using local environmental conditions plus characteristics of the material such as densities and settling velocities of the particulate matter as well as initial conditions imposed by the method of disposal, would be extremely useful. Such a model could be used not only to provide an estimate of concentrations in the receiving water (i.e. the mixing zone), but also as a valuable aid in any field monitoring program.

Purpose and Scope

4. There were two main objectives of this investigation. The first was to determine through an intensive literature search coupled with contacts with other research groups if any mathematical models currently exist for the prediction of the physical fate of dredged material disposed of in the aquatic environment. If such models were found, an assessment of their value was to be undertaken. Once the results had been obtained from this search, the second objective would be to offer the Office of Dredged Material Research (ODMR) guidance in planning research programs for future model adaption, development, and use in this area.

5. Much work has been done in the area of mathematical modeling of transport phenomena in fluids. Models by such well known researchers as Leendertse, Masch, Orlob, and Fischer, for both one- and two-dimensional geometries, have been well documented and applied to various real problems. However, such models are concerned only with water quality parameters such as dissolved oxygen (DO) and biochemical oxygen demand (BOD) and thus do not consider the transport of material containing different groups of particulate matter. Any model to be used in describing the disposal of dredged material must, of course, be capable of tracing such particulate matter through the various phases it undergoes, e.g. the convective descent and diffusive phases, until it finally settles on the bottom.

6. The major model developed in the area of physical fate prediction of dredged material is that of Koh and Chang.[2] This model allows study of three different methods of disposal of a waste material containing as many as four different types of particles, with each type possessing as many as two settling velocities. The major limitation of the model in its present form is that it was developed only for studying disposal operations in the ocean environment. Also, nothing can be said about the reliability of the model since there has been no field and very little laboratory verification of it. However, it should be stressed that the model is conceptually well designed. A detailed discussion of the model is presented in Part III of this report.

7. In addition to the discussion of the Koh-Chang model, discussions of relevant mathematical transport studies in the ocean, estuarine, and riverine environments are also presented. However, before detailed discussions of individual studies are presented, a general discussion of the processes responsible for transport phenomena in a turbulent body of fluid is presented. Appendix A presents a discussion on settling velocities of particles in a water column.

PART II: TRANSPORT PHENOMENA IN AQUATIC ENVIRONMENTS

8. There are essentially three processes responsible for the transport of a fluid property or lump of material released in a turbulent body of water. These are commonly called advection, diffusion, and dispersion. In advective processes, regular patterns of water movement carry a given property with them, thus producing a local change in its concentration. In diffusion processes, irregular movements of water called turbulence together with molecular diffusion give rise to a local exchange without any net transport of water. Quite often, the terms diffusion and dispersion are used interchangeably in the literature. However, since turbulent diffusion is associated with time averaging of the ambient fluid velocity whereas dispersion is associated with spatial averaging, the two should be differentiated. Perhaps the simplest way to illustrate the difference in the three mechanisms responsible for transport phenomena in a fluid is by means of a mathematical discussion.

Mathematical Discussion of Advection, Diffusion, and Dispersion

9. The differential equation which governs the transport of mass or a fluid property is

$$\frac{\partial c}{\partial t} + u \frac{\partial c}{\partial x} + v \frac{\partial c}{\partial y} + w \frac{\partial c}{\partial z} = D \frac{\partial^2 c}{\partial x^2} + D \frac{\partial^2 c}{\partial y^2} + D \frac{\partial^2 c}{\partial z^2} \qquad (1)$$

where

c = instantaneous concentration

u,v,w = instantaneous velocity components

D = molecular diffusion coefficient

Since it is virtually impossible to input instantaneous values of the velocities in the above equation, a time-averaging process is usually performed as discussed below.

10. Consider the velocities and the concentration to be made up of an average plus a fluctuating, i.e. turbulent, component as indicated by

$$\left.\begin{aligned} c &= \overline{c} + c' \\ u &= \overline{u} + u' \\ v &= \overline{v} + v' \\ w &= \overline{w} + w' \end{aligned}\right\} \qquad (2)$$

where $\overline{c} \equiv \frac{1}{T}\int_0^T c\ dt$, $\overline{u} \equiv \frac{1}{T}\int_0^T u\ dt$, $\overline{v} \equiv \frac{1}{T}\int_0^T v\ dt$, and

$\overline{w} \equiv \frac{1}{T}\int_0^T w\ dt$, and thus,

$$\int_0^T c'\ dt = \int_0^T u'\ dt = \int_0^T v'\ dt = \int_0^T w'\ dt = 0$$

Substituting the expressions for c , u , v , and w given by equation 2 into equation 1, integrating over some time interval T , and making use of the definitions above yields

$$\frac{\partial \overline{c}}{\partial t} + \overline{u}\frac{\partial \overline{c}}{\partial x} + \overline{v}\frac{\partial \overline{c}}{\partial y} + \overline{w}\frac{\partial \overline{c}}{\partial z} = D\frac{\partial^2 \overline{c}}{\partial x^2} + D\frac{\partial^2 \overline{c}}{\partial y^2} + D\frac{\partial^2 \overline{c}}{\partial z^2} - \frac{\partial}{\partial x}\left(\overline{u'c'}\right) - \frac{\partial}{\partial y}\left(\overline{v'c'}\right) - \frac{\partial}{\partial z}\left(\overline{w'c'}\right) \qquad (3)$$

Turbulent diffusion coefficients e_1 , e_2 , and e_3 are introduced as follows:

$$\left.\begin{aligned} \overline{u'c'} &\equiv -e_1\frac{\partial \overline{c}}{\partial x} \\ \overline{v'c'} &\equiv -e_2\frac{\partial \overline{c}}{\partial y} \\ \overline{w'c'} &\equiv -e_3\frac{\partial \overline{c}}{\partial z} \end{aligned}\right\} \qquad (4)$$

Therefore, the transport equation becomes

$$\frac{\partial \overline{c}}{\partial t} + \left(\overline{u}\frac{\partial \overline{c}}{\partial x} + \overline{v}\frac{\partial \overline{c}}{\partial y} + \overline{w}\frac{\partial \overline{c}}{\partial z}\right) = \left(D\frac{\partial^2 \overline{c}}{\partial x^2} + D\frac{\partial^2 \overline{c}}{\partial y^2} + D\frac{\partial^2 \overline{c}}{\partial z^2}\right) + \left[\frac{\partial}{\partial x}\left(e_1\frac{\partial \overline{c}}{\partial x}\right) + \frac{\partial}{\partial y}\left(e_2\frac{\partial \overline{c}}{\partial y}\right) + \frac{\partial}{\partial z}\left(e_3\frac{\partial \overline{c}}{\partial z}\right)\right] \qquad (5)$$

11. The terms inside the parentheses on the left side of equation 5 represent the advective part of the transport, while those in the parentheses on the right represent molecular diffusion. The terms inside the brackets on the right represent the turbulent diffusion portion of the overall transport. Note that these terms arise as a result of the time-averaging procedure.

12. If all problems are considered to be three-dimensional instead of simplified to either one or two dimensions, as is usually the case for most problems, there would be no dispersion terms appearing in the governing equation, i.e. equation 5. The reason for this is that dispersion terms appear as a result of spatially averaging equation 5. This is illustrated for the case of cross-sectional averaging, i.e. the problem is one-dimensional, in the following discussion.

13. Assume that the time-averaged concentration and velocity components can be represented as a cross-sectional average plus some deviation from that average, i.e.

$$\left.\begin{aligned} \bar{c} &= C + c'' \\ \bar{u} &= U + u'' \\ \bar{v} &= V + v'' \\ \bar{w} &= W + w'' \end{aligned}\right\} \qquad (6)$$

where

$$\left.\begin{aligned} C &\equiv \frac{1}{A}\int_0^A \bar{c}\, dA \\ U &\equiv \frac{1}{A}\int_0^A \bar{u}\, dA \\ V &\equiv \frac{1}{A}\int_0^A \bar{v}\, dA = 0 \\ W &\equiv \frac{1}{A}\int_0^A \bar{w}\, dA = 0 \end{aligned}\right\} \qquad (7)$$

in which A is the cross-sectional flow area; thus,

$$\int_0^A c'' \, dA = \int_0^A u'' \, dA = \int_0^A v'' \, dA = \int_0^A w'' \, dA = 0$$

Substituting equation 6 into equation 5, integrating over the cross section, neglecting the molecular diffusion terms, and assuming uniform flow yields

$$\frac{\partial C}{\partial t} + U \frac{\partial C}{\partial x} = \frac{\partial}{\partial x}\left(e_1 \frac{\partial C}{\partial x}\right) - \frac{\partial}{\partial x}\left(\widetilde{u''c''}\right) \tag{8}$$

The longitudinal dispersion coefficient E is now introduced as

$$\widetilde{u''c''} \equiv - E \frac{\partial C}{\partial x} \tag{9}$$

Using the above definition, equation 8 becomes

$$\frac{\partial C}{\partial t} + U \frac{\partial C}{\partial x} = \frac{\partial}{\partial x}\left(e_1 \frac{\partial C}{\partial x}\right) + \frac{\partial}{\partial x}\left(E \frac{\partial C}{\partial x}\right) \tag{10}$$

where

$U \frac{\partial C}{\partial x}$ = advection

$\frac{\partial}{\partial x}\left(e_1 \frac{\partial C}{\partial x}\right)$ = turbulent diffusion

$\frac{\partial}{\partial x}\left(E \frac{\partial C}{\partial x}\right)$ = dispersion portion of the total longitudinal transport

14. Since motion in large bodies of water such as estuaries or the ocean consists of a more or less continuous spectrum of scales ranging from the molecular free path up to the oscillatory motion of tides in estuaries or to the oceanwide general circulation in oceans, a division between the part of the motion assigned to the advective process and the part that leads to the turbulent diffusion process is difficult to make. It is quite easy to see that the term "velocity" has little meaning unless accompanied by some indication of the mode of averaging employed. The components of motion that occur on a scale smaller than that used in the velocity-averaging process do not appear in the average velocity field.

Diffusion and Dispersion Coefficients

15. From the discussion above concerning velocity averaging, it should be quite obvious that the diffusion coefficients are dependent upon the time scale used in the averaging process. These coefficients are often called eddy diffusivities, whereas the dispersion coefficient in the one-dimensional case is known as the longitudinal dispersion coefficient.

16. Equation 1 with constant coefficients is the classical Fickian equation of diffusion. In the ocean, eddies much larger than the diffusing cloud advect it as a whole, while eddies much smaller produce a scattering of the cloud about its center of mass by turbulent diffusion. As the cloud grows in size, the boundary between the eddies contributing to advection and those responsible for diffusion shifts toward larger scale. Since the energy contained within the eddies increases with eddy size, the apparent power of mixing increases with patch size. Therefore, a model based upon the Fickian diffusion equation is inappropriate for ocean mixing.

17. Stommel[3] first showed that the Richardson equation describing atmospheric diffusion also describes horizontal diffusion in the ocean with the same "4/3 power law." This law states that the variation of the horizontal coefficient of eddy diffusivity K with the scale of the turbulent phenomenon ℓ is

$$K = e_1 = e_3 = a\varepsilon^{1/3}\ell^{4/3}$$

where a is a proportionality constant and ε is the rate of energy dissipation. This 4/3 law has been reconfirmed by, among others, Orlob.[4] However, the determined proportionality constant, as noted by Okubo and Pritchard,[5] varies among investigators not only in values ranging from 0.002 to 0.05 $cm^{2/3}$/sec, but also with observational conditions including a range of scale. According to the computation of the horizontal coefficient of diffusivity by Gesenzwei,[6] the dependence of the coefficient on the averaging time is given by

$$K = m\varepsilon^{1/3}\nu^{2/3}T^{2/3}$$

where

m = nondimensional proportionality constant

ν = molecular viscosity

This compares with $K = a\varepsilon^{1/3}\ell^{4/3}$, if $(\nu T)^{1/2}$ has the meaning of the scale of the phenomenon.

18. The vertical eddy diffusion coefficient e_2 is strongly dependent on the vertical density gradient and thus does not follow a 4/3 law such as the horizontal coefficient does. A semiempirical study based on the mixing length theory was developed by Rossby and Montgomery.[7] They determined that

$$e_2(z) = e_{2_n}(z)\left(1 + \beta R_i\right)^{-1}$$

where

e_{2_n} = vertical diffusion coefficient in the case of no density gradient (neutral case)

β = undetermined proportionality constant

R_i = Richardson number given by $R_i = [(g/\rho)(\partial\rho/\partial z)]/(dv/dz)^2$, in which g is gravitational acceleration and ρ is the fluid density

Other investigators such as Holzman,[8] Yamamoto,[9] and Mamayev[10] have proposed different forms for e_2, with Mamayev proposing the exponential form

$$e_2 = e_{2_n} e^{-\beta R_i}$$

19. As would be expected, many investigators have experimentally determined numerical values for diffusion coefficients. Folsom and Vine[11] followed the spread of a radioactive tracer in the ocean over a horizontal area of 40,000 km^2 over a period of 40 days. During this time, vertical mixing occurred through only 60 m. Eddy diffusivities on the order of 10^7 cm^2/sec in the horizontal and 1 cm^2/sec in the vertical were determined. Koh and Chang[2] present tables of the numerical values

various investigators at several locations in the world's oceans and estuaries have obtained for both the horizontal and vertical eddy diffusion coefficients.

20. The first important study of longitudinal dispersion in turbulent shear flow was by Taylor[12] for the case of flow in a pipe. Elder[13] found that for an infinitely wide, open channel an expression for the longitudinal dispersion coefficient is given by $E = 5.9dU^*$, where d is the channel depth and U^* is the shear velocity. Fischer[14] has shown that in natural streams longitudinal dispersion is accomplished almost entirely due to lateral variation in the fluid velocity. In such streams, there is considerable variation of velocity between transverse positions; e.g. there is usually a high-velocity zone either in the center or near one bank and other zones of lower velocity. Since material in the high-velocity zone is carried downstream faster than that in the low-velocity zones, the effect is a stretching out of the cloud. In his study, Fischer found that the longitudinal dispersion coefficients for various natural streams varied from 50 to 700 rU^* (r is the hydraulic radius).

21. It should be noted that the above discussion has been for diffusion of patches of material, such as fluorescent dye, which are visible in water. However, when considering the diffusion of material such as dredge spoil, the concern is with the transport of various solid particles characterized by different settling velocities. A logical question would seem to be: What effect does this have on the diffusion coefficients? In other words, can diffusion coefficients determined from dye studies be used in diffusion studies of dredged material containing particulate matter?

22. The assumption is often made that

$$\varepsilon_s = \beta_1 \varepsilon_f$$

where

ε_s = sediment turbulent mass transfer coefficient

β_1 = a constant

ε_f = fluid turbulent mass transfer coefficient

Most previous investigators, such as Carstens,[15] have concluded that $\varepsilon_s < \varepsilon_f$ because particles do not respond fully to turbulent velocity fluctuations. Singamsetti[16] studied the diffusion of sediment in a submerged water jet and is one of the few investigators to conclude that $\varepsilon_s > \varepsilon_f$. He reasoned that, in a turbulence composed of vortices, the centrifugal force acting on sediment particles would be greater than that acting on fluid particles. Due to the greater centrifugal force, the solid particles would be thrown to the outside of the eddies, and this action would subsequently result in an increased rate of diffusion. Jobson and Sayre[17] present a discussion in which they attempt to clarify the apparent contradictions of investigators such as Carstens and Singamsetti. In general, it appears that values of eddy diffusion coefficients obtained from dye studies are used in most diffusion studies of particulate matter.

PART III: MODELS AND RELEVANT STUDIES APPLICABLE TO OCEAN DISPOSAL

23. The ocean can be represented as an upper layer with a depth ranging from 10 to a few hundred meters, a thermocline region where density gradients can be quite large, and a deep layer of cold, dense water. The upper layer is characterized by nearly homogeneous mixed waters within which are found the predominant currents, turbulent motion, and seasonal variations in temperature, salinity, and DO. The general features of the mean navifacial current patterns of the oceans are relatively well recognized. Most strong currents are found along the western boundaries of the oceans, e.g. the Gulf Stream in the North Atlantic. In the interior area away from these strong currents, current speeds are on the order of 10 cm/sec. The major external forces acting on the surface currents are the prevailing wind systems. However, the circulation patterns are also influenced by the earth's rotation, the density distribution in the water, and the ocean boundaries. The direct influence of wind on water movements is usually limited to approximately the upper 200 m. Currents in the deep layer seem to be closely related to the density distribution.

24. Any analysis of waste disposal in the ocean as well as in other aquatic environments should include determinations of the concentration of the waste material in suspension and in solution and the distribution of disposed solids, either floating at the surface or settled on the ocean floor. Given the waste characteristics, the ocean environmental conditions, and the method of disposal, Koh and Chang[2] have developed a mathematical model capable of providing essentially all of the required information discussed above.

Koh-Chang Mathematical Model for Prediction of Dispersion and Settling in Barged Ocean Disposal of Wastes

25. In the disposal of waste material in the ocean environment, the primary concern is with dumping from barges. In some disposal

operations, the dumping of the material is accomplished by instantaneously releasing the material from the bottom of the barge. In other cases, disposal is accomplished by pumping the material through a pipe under the barge while the barge is moving or, in the case for which wide dispersion is sought, by releasing the material in the barge wake. The mathematical model developed by Koh and Chang has the capability of handling all of the disposal operations listed above. The basic assumption upon which the model is based (which makes it especially applicable to the disposal of dredged material) is that the waste material consists of two phases, a solid phase and a liquid phase. Furthermore, the solid phase is assumed to consist of a discrete set of solid particle densities, fall velocities, and concentrations, while the liquid phase is assumed to be miscible with the ambient water.

Disposal operation 1:
instantaneous bottom dumping

26. The first barge disposal operation considered is simple bottom dumping in which the release is assumed to be essentially instantaneous. The waste material is assumed to undergo a possibility of three different phases: convective descent, dynamic collapse (includes the possibility of bottom encounter), and long-term diffusion. It should be noted that in some cases the dynamic collapse phase may be bypassed.

27. In the description of the convective descent phase, equations expressing conservation of mass, momentum, buoyancy, and vorticity of the waste cloud plus conservation of solid particles are formulated. The characteristics of the element are assumed similar at all stages of its motion, and the cloud is assumed to retain the shape of a hemisphere. While undergoing this phase, the waste cloud usually gains a significant amount of mass and momentum through entrainment, which of course is accounted for in the conservation equations. In order to obtain a solution of the above equations, several coefficients, e.g. entrainment and settling coefficients, must be specified before a description of the motion of the waste cloud can be obtained. The model contains suggested values for most of these, but the user does have the option of external specification. During this initial phase of convective descent, those

particles with fall velocities greater than the descent velocity of the cloud settle out and are immediately subjected to the long-term diffusion phase. The convective descent phase continues until the cloud reaches a neutrally buoyant position due to the increasing ambient density and the loss of heavy particles.

28. In actuality, its momentum tends to overshoot the convective element beyond the neutrally buoyant position, while the buoyancy force tends to bring the cloud back; thus, the cloud tends to oscillate about this position. While the gross vertical motion of the cloud is largely suppressed, the cloud tends to collapse vertically and spread out horizontally seeking a hydrostatic equilibrium with the ambient fluid. This dynamic collapse phase is a result of the density gradient within the cloud being different from that of the ambient. The basic conservation equations governing the convective descent are also applied to the dynamic collapse phase. However, as the cloud collapses, more dimensions are needed to completely describe the motion. These dimensions are incorporated through the assumption that the cloud cross section retains an ellipsoidal shape. Once again, as in the convective descent phase, various coefficients must be specified in order to obtain a solution of the governing equations. The initial conditions which must be input for this phase are obtained from the solution of the equations governing the convective descent phase.

29. If the density stratification is not strong enough, the waste cloud will ultimately hit the bottom. With the incorporation of two additional forces, a reaction and friction force at the bed, the equations used in the dynamic collapse phase can be used again since the motion in the bottom encounter phase is very similar to that which occurs in the dynamic collapse phase. The vertical motion is suppressed by the bottom, and the cloud essentially undergoes only horizontal spreading. A solution of the governing equations is obtained by using the solution of the convective descent phase as the required initial conditions.

30. The convective descent phase is either terminated by bottom encounter or by the cloud reaching its position of neutral buoyancy. The dynamic collapse phase terminates when the estimated horizontal

spreading due to diffusion is larger than that due to dynamic collapse. As solid particles settle from the cloud during the convective descent and dynamic collapse phases, they are subjected to long-term diffusion. At the end of the dynamic collapse phase, all of the remaining waste material is input into the diffusion phase.

31. The turbulent diffusion equation previously discussed, with an additional term to account for the settling of solid particles W_s, as shown below, is transformed and then becomes the basic equation governing the transport of the waste material during the long-term diffusion phase.

$$\frac{\partial \bar{c}}{\partial t} + \bar{u}\frac{\partial \bar{c}}{\partial x} + \bar{v}\frac{\partial \bar{c}}{\partial y} + \bar{w}\frac{\partial \bar{c}}{\partial z} = \frac{\partial}{\partial x}\left(e_1 \frac{\partial \bar{c}}{\partial x}\right) + \frac{\partial}{\partial y}\left(e_2 \frac{\partial \bar{c}}{\partial y}\right) + \frac{\partial}{\partial z}\left(e_3 \frac{\partial \bar{c}}{\partial z}\right) - \frac{\partial}{\partial y}(W_s \bar{c}) \qquad (11)$$

Boundary conditions for the fluid portion of the waste, the sinking particles, and the floating material must be specified at both the surface and the ocean floor. The boundary conditions for the solid particles take into account that material is being stored on the boundary as well as the fact that some material becomes reentrained after settling. The initial conditions for this phase are obtained from the results of either the convective descent or the dynamic collapse phase, depending on whether or not the dynamic collapse phase is bypassed.

32. As previously mentioned, the diffusion model employed is equation 11 transformed by the Aris Method of Moments. With such a model, the detailed distribution of the material is ignored, and only the gross characteristics of the dispersant as functions of time and depth are determined. This transformation is accomplished by multiplying each term in equation 11 by $x^k z^\ell$, in which $k = 0, 1, 2, \ldots$, and $\ell = 0, 1, 2, \ldots$, and then integrating over the horizontal plane. The boundary conditions must be similarly treated. Once the above integration is completed, x and z are eliminated as independent variables. As noted, the penalty for such a transformation is that a detailed

distribution is unattainable since only the moments of the horizontal distribution of waste material rather than the concentration itself can be found. The first three moments, i.e. $k = 0, 1, 2$, and $\ell = 0, 1, 2$, provide the necessary information for determining the total volume under the concentration curve, the average displacements of the distribution centroid, and the variances of the distribution. Additional limitations of the model as a result of the Aris transformation will be discussed later.

33. A detailed list of the input required for disposal operation 1 of the model is as follows:

a. Ambient density profile. Up to 30 points are allowed.

b. Vertical eddy diffusion coefficient profile. The fixed profile requires three coefficient values and four corresponding depths. Fig. 1a illustrates this.

c. Number of prototype seconds the program is to simulate.

d. New time steps instead of those specified by the program if the previous trials indicate a need.

e. Initial velocity of the waste cloud.

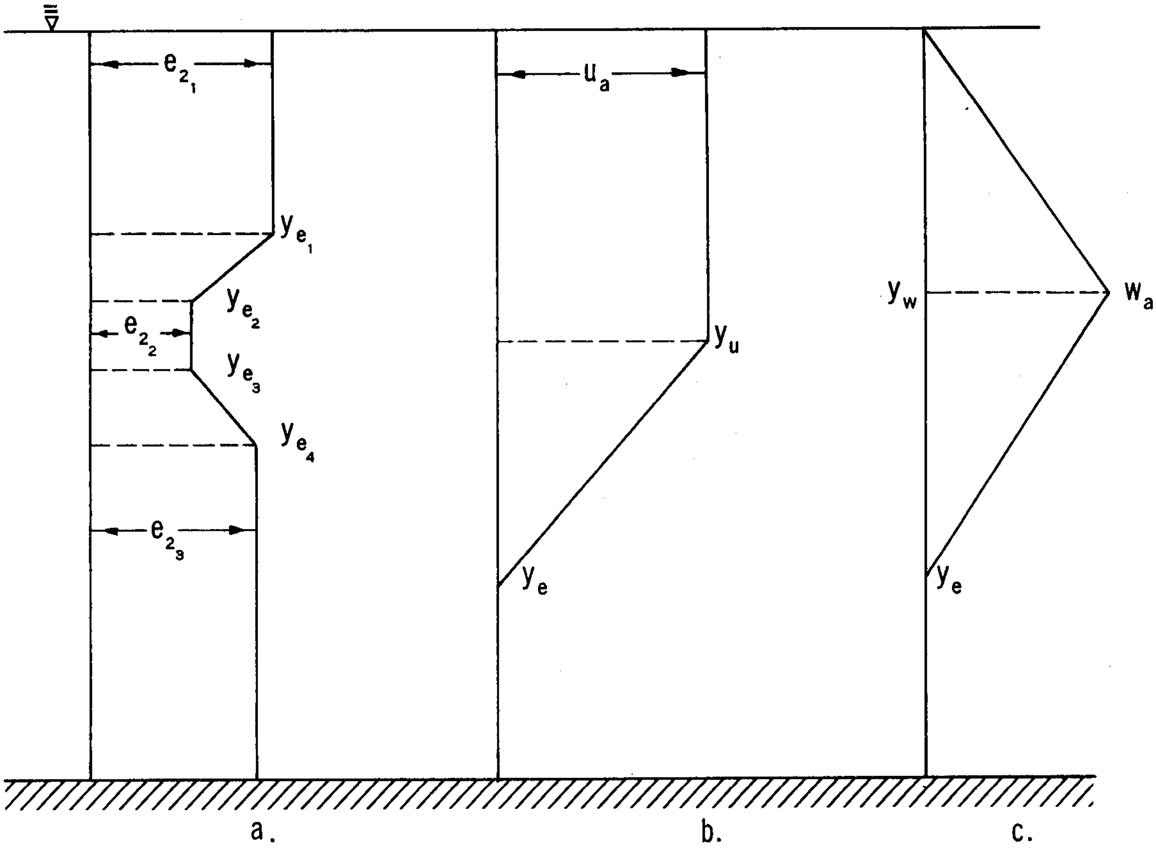
f. Number of the phase after which the program terminates.

g. Control for whether or not fluid concentration is given in the output.

h. Ambient current profiles, which must be specified as shown in figs. 1b and 1c, i.e. one value of the current and two depths for both horizontal currents.

i. Particulate material densities, concentrations, and settling velocities. Up to four densities and two settling velocities per density may be input.

34. Output given at the end of the convective phase consists of the cloud centroid coordinates, density difference between cloud and ambient, cloud radius, solid particle concentration, fluid concentration, and volume of solid particles. Similar information is output in the dynamic collapse phase. At the end of the long-term diffusion, at particular points in time, the output consists of x, y, and z coordinates of the centroid, the variances, and the volume of solids in horizontal plane versus depth. The x and z coordinates and variances, as well as the volume of solid material deposited on the bed, are also given.



a. Vertical eddy diffusion coefficient profile

b. Horizontal component of ambient velocity in x direction

c. Horizontal component of ambient velocity in z direction

Fig. 1. Ambient conditions for long-term diffusion model

A form for the distribution such as uniform or Gaussian must be assumed in order to determine the height of material deposited on the bed.

Disposal operation 2: jet discharge

35. The second barge disposal operation which the Koh-Chang model can handle is that of releasing the dredged material through a pipe under a moving barge, either by pumping or gravity dump. Therefore, in order to analyze such a disposal scheme, the study must be concerned with mixing phenomena in jets. Perhaps the most elaborate mathematical model of mixing in a buoyant jet is that developed by Hirst.[18] A detailed discussion of Hirst's work is given in paragraphs 47-51 in connection with the Edge-Dysart model. Other investigators who have

considered the problem of mixing in buoyant jets and plumes include Albertson et al.,[19] Abraham,[20] Morton et al.,[21] Fan,[22] Fox,[23] and Hoult et al.[24] As previously noted, Singamsetti[16] has investigated the diffusion of sediment in a submerged jet. In the Koh-Chang model, Abraham's and Fan's approaches are extended to study a sinking jet containing sediment in a density-stratified, nonuniform, two-dimensional crossstreamflow pattern near the nozzle. Similar to the case of simple bottom dumping, i.e. disposal operation 1, the jet undergoes both a convective and a collapse phase before entering long-term diffusion.

36. During jet convection, it is assumed that the jet cross section remains circular. Initially, top hat velocity, density, and concentration distributions of the waste material are assumed. The jet can flow in any direction depending on its initial momentum and the ambient current. The governing partial differential equations representing the conservation of mass, momentum, etc., are developed in a coordinate system which moves with the jet center line. These equations contain no time dependence and are simplified further by being integrated over a cross section of the jet. The resulting equations governing the motion of the jet constitute a set of ordinary differential equations which can be readily solved. Again, it should be noted that the solid particles tend to settle out, and thus settling coefficients must be specified. Other coefficients include entrainment coefficients and those associated with friction and drag forces. The entrainment function consists of two terms, one applying to momentum jets and the other to a two-dimensional thermal. Some discussion of other entrainment functions found in the literature will be presented later.

37. When the jet plume is far downstream from the nozzle, it no longer behaves like a jet. Rather, its behavior is similar to a two-dimensional, elliptically shaped thermal. The plume tends to collapse vertically and spread out horizontally seeking a hydrostatic equilibrium in the ambient density gradient. With the above assumption, the equations expressing conservation of mass, momentum, etc., are formulated and may be solved using the solution from the jet convective phase as initial conditions.

38. Similar to the disposal by simple bottom dumping, the waste material plume can reach the bottom and spread out if the ambient density stratification is not strong enough to arrest the vertical descent of the plume. With the assumption that at bottom encounter the cross section of the plume has a half-elliptical shape, the governing equations can once again be formulated and solved employing initial conditions obtained from the solution of either the jet convective or dynamic collapse phases.

39. The diffusion model employed in disposal operation 1 also governs the transport of material during the long-term diffusion phase of the jet discharge method of disposal. Once again, as in bottom dumping, as the heavier particles settle out during the convective and dynamic collapse phases, they immediately enter long-term diffusion. At the completion of the dynamic collapse phase, the remaining waste material is input into the diffusion model.

40. The output furnished and the input required by the operation of the jet discharge portion of the model are very similar to the input and output of bottom dumping. An example of additional input is that the initial orientation of the jet must be specified.

Disposal operation 3:
<u>release in the barge wake</u>

41. The Koh-Chang model also allows for the disposal of waste material into the wake of a moving barge. This method of disposal would be useful in the disposal of waste material in which wide dispersion is desired. The material undergoes an initial mixing phase in which it is assumed that, because of the strong turbulent mixing, the buoyancy effect is of secondary importance. This initial mixing is accounted for through empirical expressions derived by Naudascher. After the initial mixing, the buoyancy forces the half-cylinder waste material plume to descend vertically to seek a neutrally buoyant position while it is convected downstream by the ambient current. This convective descent phase is then followed by dynamic collapse and long-term diffusion similar to those processes encountered by the material in the other disposal operations. Although in theory this portion of the model could be applied to dredged

material, no additional discussion is presented here since from a practical standpoint it is highly unlikely that dredged material will be disposed of in this manner.

Limitations of
the Koh-Chang model

42. From a conceptual standpoint, the Koh-Chang model is an extremely well developed and potentially useful model for determining the physical fate of dredged material disposed of in the ocean. However, there are inherent limitations which restrict the widespread application of the model. In addition, many applications of the model will be required in order to obtain representative values for the various coefficients which must be specified.

43. The major limitation of the model that strictly limits its applicability to disposal in an ocean environment is its representation of the ambient current. The ambient current is composed only of horizontal components, which, furthermore, must take the form shown in figs. 1b and 1c. In addition, these currents can only be a function of the depth coordinate, and thus no horizontal or time variation is allowed. This limitation could not easily be removed since it is essential in the development of the diffusion model based upon the Aris moment method. In order to apply the model to a more dynamic environment, such as an estuary in which spatial as well as time variation of ambient currents due to tidal actions may occur, it appears that the Aris method needs to be discarded.

44. Another limitation of using the Aris method lies in the area of comparing recorded field data with computed results from the diffusion phase. Field measurements for most dispersion studies are usually taken at a particular spatial point and are for concentration versus time, whereas the computed results from the diffusion equation transformed by the Aris method consist of moments of the distribution of waste material in horizontal planes at a particular time. This may not be a serious limitation since the horizontal plane containing the greatest amount of material or deposition of bottom material rather than concentrations at some point in the water column versus time will probably be

used for comparison. It should be noted that the question of what to use for model verification is an important one and will require considerable thought before field experiments can be designed.

45. An additional limitation of the model lies in the neglection of a vertical ambient velocity. As Koh notes, in the ocean, this is a valid assumption; however, in an inland lake or bay with lateral boundaries, continuity would dictate that the vertical velocity could not be zero everywhere. It should also be noted that in addition to the limitation above, lateral boundaries would probably prohibit the use of the Aris method.

46. Although it should not be construed as a limitation, one possible area of improvement in the model might be in the representation of the entrainment function employed in the jet discharge disposal operation. As noted by Hirst,[18] the entrainment function should depend on the following:

- a. Local mean flow conditions within the jet.
- b. Local buoyancy within the jet.
- c. Velocity ratio of the jet and ambient current.
- d. Initial jet orientation.
- e. Ambient turbulence.

It appears from an inspection of the entrainment function employed that conditions a, c, and d have been considered. In comparison, Hirst formulates an entrainment function that is dependent upon conditions a, b, c, and d. Entrainment functions postulated by other researchers and a discussion on which of the above conditions are taken into account are presented by Hirst.[18]

Edge-Dysart Model for Barge-Released Dredged Material

47. As with the Koh-Chang model, the Edge-Dysart model[25] was developed for barge dumping into the ocean environment. The only disposal operation considered by the model is that of jet discharge with the assumption that the jet remains in a single plane. The material is assumed to undergo only two phases, jet convection and long-term diffusion,

whereas the Koh-Chang model allows for an intermediate dynamic collapse phase plus the possibility of bottom encounter. Based upon results from various applications, Koh and Chang found that the results from their long-term diffusion model were changed by an order of magnitude when the dynamic collapse phase was ignored.

48. The integral conservation equations governing the jet convective phase are taken from Hirst with the assumption that the ambient current is negligible compared with the jet speed. As previously discussed, the governing partial differential equations are developed in a coordinate system which moves with the jet center line. The equations are then simplified by assuming the flow to be axisymmetric and are next integrated over a cross section of the jet. The assumptions implicit in the derivation of the integral equations given by Hirst are:

- a. The flow is steady.
- b. The flow is fully turbulent. Molecular diffusion is neglected.
- c. The fluid is assumed incompressible. Density variations are included only in the buoyancy term.
- d. All other fluid properties are assumed constant.
- e. Fluid velocities are low enough so that frictional heating can be neglected.
- f. The pressure variation is purely hydrostatic.
- g. Changes in density are small enough so that a linear equation of state is valid.
- h. Flow within the jet is of the boundary layer type, and the boundary layer approximations are valid.
- i. Flow within the jet is axisymmetric.
- j. The jet is discharged to an infinite fluid of infinite extent.

Hirst assumes initial velocity and density profiles that are Gaussian and, as previously discussed, formulates an entrainment function which includes the effects of internal turbulence, buoyancy, and crossflows. However, it should be noted that the entrainment function used by Edge and Dysart[25] is dependent only upon the local mean flow conditions within the jet.

49. The jet convective phase terminates when the level of neutral

buoyancy, which is assumed always to occur far enough above the ocean floor so that bottom encounter never occurs, is reached. As the material settles from the level of neutral buoyancy it is transported in the horizontal directions. The scale of turbulent diffusion in the vertical direction is considered to be much smaller and is thus neglected.

50. The long-term diffusion model consists of utilizing the form determined by Orlob[4] for the horizontal diffusion coefficient, i.e.

$$K = 0.00016\ell^{4/3}$$

where $\ell = 4\sigma$ and σ is the standard deviation of material in the moving, spreading patch. Equating the above expression to the expression

$$K = \frac{1}{2}\frac{d\sigma^2}{dt}$$

and letting $\sigma = \sigma_b$, which is the standard deviation at the level of neutral buoyancy, yields

$$\sigma = \sigma_b \left(1 + 4^{2/3}\,\frac{2}{3}\,\frac{0.00016t}{\sigma_b^{2/3}}\right)^{2/3}$$

in which t is the time after the cloud has moved from the neutrally buoyant level. Different settling velocities are handled for groups of particles by applying the appropriate time in the above expression. With a knowledge of the distribution of material with settling velocity and a knowledge of the waste characteristics, the total accumulation of material can be obtained by using the above expression for σ in a Gaussian distribution.

51. From the above discussion, it should be obvious that the Edge-Dysart model is similar to that portion of the Koh-Chang model concerned with disposal by jet discharge. However, it should also be obvious that the latter contains a much better treatment of the long-term diffusion and provides for more detailed tracing of the waste cloud, e.g. the dynamic collapse and bottom encounter phases. Therefore, if one or the

other were to be selected for implementation, the Koh-Chang model would be the natural selection.

Additional Ocean Dispersion Studies

52. Guided by results from dye studies, various investigators, including Joseph and Sendner, Ozmidov, and Schönfeld (as discussed by Okubo[26,27]), have developed theoretical models of turbulent diffusion in the ocean. These studies are useful from the standpoint of providing a better understanding of diffusion in the ocean; however, they are of little use in determining the physical fate prediction of a multiphase waste such as dredged material possessing initial buoyancy and momentum as a result of the disposal operation.

53. Ketchum and Ford[28] were concerned with the case of a moving barge discharging iron particles into its wake. In their study, vertical dispersion was considered to be instantaneous, and horizontal diffusion along the axis of the wake was neglected. By treating the problem as though all of the iron discharged was concentrated along the wake median line at time $t = 0$, plus making the assumption that the distribution of waste along lines perpendicular to the axis is Gaussian, a simple expression for the concentration, using a mixing coefficient independent of the dimensions of the mixing field, was obtained.

54. Many studies of the discharge of waste material from ocean outfalls, e.g. the studies previously mentioned involving mixing in jets and plumes, have been undertaken. Environmental Science and Engineering, Inc., has been extensively involved in such studies for cities such as Pompano Beach and Hollywood on the Florida coast.[29] The model they have employed is a generalized Gaussian distribution model for predicting concentrations from a continuous point source in a homogeneous turbulence. Letting S_y and S_z represent standard deviations in the lateral and vertical directions, the mean concentration $N(x,y,z)$ at any point downstream from the source is given by

$$N(x,y,z) = \frac{Q}{\pi \bar{U} S_y S_z} \exp\left[-\frac{1}{2}\left(\frac{Y}{S_y}\right)^2 - \frac{1}{2}\left(\frac{Z}{S_z}\right)^2\right]$$

where

Q = amount of waste released per unit time corrected to include the effects of decay

$\bar{U}$ = mean current velocity within the waste field

The standard deviations are assumed to be functions of the diffusion time or distance from the source and are estimated from dye-plume concentration data. Once again, such simplified models appear to have little applicability to disposal operations encountered with multiphase dredge material.

PART IV: TRANSPORT STUDIES RELEVANT TO ESTUARINE DISPOSAL

55. Compared with the current systems in the open ocean, current patterns in estuaries and coastal areas are far more complex because of the role which tidal forces, local winds, and forces due to local density differences play in the determination of currents. The horizontal density gradient, due to the increase in salinity from the head of the estuary toward its mouth, gives rise to horizontal pressure gradients which vary with depth. These tend to produce a circulation in which the fresh water flows seaward as a layer overlying salt water of sea origin. Natural estuaries are generally classified as either tidal-mixed or nontidal-mixed with various subclassifications such as frictionless, well-mixed, partially stratified, etc., as illustrated in fig. 2 taken from Glenne.[30]

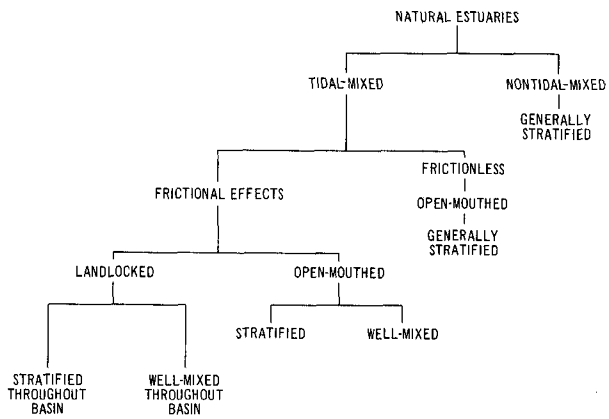


Fig. 2. Estuary classification diagram

56. In the estuarine environment, due to the strong oscillatory time-dependent current structure, the role of convection becomes very important in the transport of mass, whereas in the ocean environment turbulent diffusion is the dominant transport mechanism. The fate of a pollutant introduced into a tidal estuary is dependent upon the method of introduction, the relative density of the undiluted effluent with respect to the density of the receiving waters, the vertical variation in density in the estuary, the strength of the tidal currents, and the intensity of the turbulent diffusion process. It should also be noted

that velocity shears, the effects of which are accounted for through dispersion coefficients if spatial averaging is employed, contribute to the scattering or dispersion of the pollutant.

57. The steady state features of a jet or plume in a field of constant velocity cannot be applied to an estuary. Evolving plumes from continuous releases in estuaries and coastal waters show the following feature: The oscillating tidal currents whip the growing plume back and forth about the source so that, at any time, only one plume may be visible, the front of which advances only to the limit of the tidal excursion. The old plumes which existed prior to that time have been detached from the source before they become so diluted by dispersion that they only participate in building the background concentration.

58. Perhaps at this point it should be noted that no mathematical models have been found that appear to be capable of accurately tracing dredged material disposed of in an estuarine environment from its initial release into the water column until it becomes stored on the estuary bottom. The water quality models discussed below are presented solely because the basic building block for them is some form of the advection-diffusion equation just as it is for the dredged material disposal problem. They are not discussed because they have any direct application to the dredge spoil problem. Extensive modification of the Koh-Chang model could perhaps result in it being applicable to estuarine disposal. This modification would center around allowing time and spatial dependence of the ambient current in the particulate diffusion model.

Estuarine Water Quality Dispersion Models

59. As noted by Orlob,[31] four investigating teams have been continuously prominent for several years in the area of mathematical modeling of tidal hydraulics and estuarine water quality parameters. These are:

- a. R. V. Thomann, D. J. O'Connor, and their associates of Manhattan College, New York, and Hydroscience, Inc., of Leonia, New Jersey.

b. G. T. Orlob, R. P. Shubinski, and their co-workers of Water Resources Engineers, Inc., of Walnut Creek, California, and Springfield, Virginia.

c. F. D. Masch and his associates at the University of Texas.

d. Jan Leendertse of the Rand Corporation, Santa Monica, California.

60. Thomann and his associates developed a finite segment model of the Delaware River Estuary. Conceptually, the Delaware model treats the estuary as a linear system of 30 discrete segments within which the flow is regarded as unidirectional and steady. Mass balance equations for the DO concentration and the BOD are written for each segment. These comprise the mathematical model of the estuary as formulated for the Delaware. Some basic assumptions inherent in the Delaware model are:

a. Flow is steady for the season of interest.

b. Velocities are unidirectional in accordance with net hydrologic balance for each segment.

c. Mixing due to tidal effects or other unsteadiness in the flow may be considered as random, i.e. diffusional.

d. Complete mixing is assumed for each segment.

61. The Bay Delta Models developed by Orlob, Shubinski, and their associates at Water Resources, Inc., consist of a hydrodynamic as well as dynamic and steady state water quality models. Each of the three models is structured conceptually for a particular application as a one-dimensional network approximation of a shallow, fully mixed system of interconnecting channels, i.e., branching and looping are permitted in the discretized system. The dynamic water quality model is formulated from the basic advection-diffusion equation written for each quality constituent modeled. A major simplification of the dynamic model results from dropping the diffusional term. Orlob indicates that experience in estuaries where the branching and looping network conceptualization (node-link) has been applied dictates that, when dynamic conditions are well represented and large numbers of discrete elements are used to describe the hydrodynamic behavior, the diffusion term is negligible. The steady state model is essentially the same, except the assumption is made that for a given set of steady flows there is no net change with

time in the mass of each constituent. It should be noted that the diffusion term is not dropped in the steady state model, and thus eddy diffusion coefficients are required.

62. Both the water quality model developed by Masch and that developed by Leendertse are vertically averaged two-dimensional models applicable to well-mixed estuaries and coastal seas. Both models are formulated on finite difference grids, although Leendertse uses an alternating direction implicit-explicit technique for solution of the set of quality equations, whereas the finite difference representation employed by Masch is completely implicit. The models of both investigators use values obtained from hydrodynamic models for the ambient velocities which must be input. Leendertse's model is so closely tied to his hydrodynamic model that in essence it is an extension of it.

63. Fischer[32] developed a one-dimensional model, which allows for branching, that is capable of predicting the movement and dispersion of a pollutant in a tidal embayment. Each time step includes a convective step, a diffusive step, and a concentration decay step. The program can generate the motion and dispersion of a concentrated slug of pollutant, or it can predict concentrations resulting from a continuous discharge at a particular point. The program is Lagrangian in concept in that attention is fixed on the motion of identified finite elements of fluid. Diffusion is permitted between the elements. Numerical dispersion, which occurs in the convective step if concentrations must be assigned to fixed points on a spatial grid, is minimized in this model because no spatial grid is established. Instead, the exact location of each finite element is computed.

64. A comprehensive review and bibliography of one- and two-dimensional models of estuarine hydrodynamics and transport phenomena may be found in a report by the Environmental Protection Agency (EPA).[33] Again it should be noted, however, that the models discussed in the EPA report as well as the Masch, Leendertse, and other such water quality models discussed here were not developed to handle the transport of a multiphase waste material containing solid particles settling at different rates.

Van de Kreeke's Model for Pipeline Discharge of Dredged Material

65. Van de Kreeke[34] has attempted to mathematically model the discharge of dredged material from a pipeline located below the water surface during periods of definite ebb and flood currents in an estuary. The total discharge was approximately 10 lb* of solids per second, and most of the material appeared to be in the silt range. About 90 percent of the particles were smaller than 85 μ, and 10 percent were smaller than 10 μ.

66. With the assumptions:

- a. The convective velocity $u(t)$ is constant over the depth;
- b. Lateral and longitudinal diffusion are negligible;
- c. The vertical eddy diffusivity coefficient is constant;
- d. A block of polluted water travels at the average current velocity $u(t)$, and thus the longitudinal position x of the block at time t is $x = u \cdot t$;
- e. All particles reaching the bottom remain there, i.e. no reentrainment;
- f. The fluid properties are not altered by the suspended sediment;
- g. The sediment concentration in the vertical at the disposal site is constant; and
- h. No flocculation occurs;

the governing diffusion equation becomes

$$\frac{\partial c}{\partial t} = \varepsilon \frac{\partial c^2}{\partial y^2} + W_s \frac{\partial c}{\partial y}$$

The boundary conditions to be satisfied are

$$\text{at } y = 0 ; \quad \varepsilon \frac{\partial c}{\partial y} = -\lambda$$

$$\text{at } y = h ; \quad \varepsilon \frac{\partial c}{\partial y} = -W_s c$$

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* A table of factors for converting British units of measurement to metric units is presented on page xiii.

where λ is the rate of erosion and h is the average channel depth. In addition, initially at time $t = 0$, $c = C_o$.

67. To check the validity of the model, a comparison between measured and computed concentrations was made by Van de Kreeke. The parameter values and initial conditions input into the model were:

$$C_o = 1250 \text{ ppm}$$
$$h = 10 \text{ ft}$$
$$\varepsilon = 0.068\ hU^* = 0.05\ \text{ft}^2/\text{sec}$$
$$W_s = 0.008 \text{ ft/sec}$$

and the background sediment concentration was assumed to be 100 ppm in the determination of λ . A comparison of measured and computed suspended sediment concentrations at two depths and four longitudinal distances from the source is presented in fig. 3. Note that within a

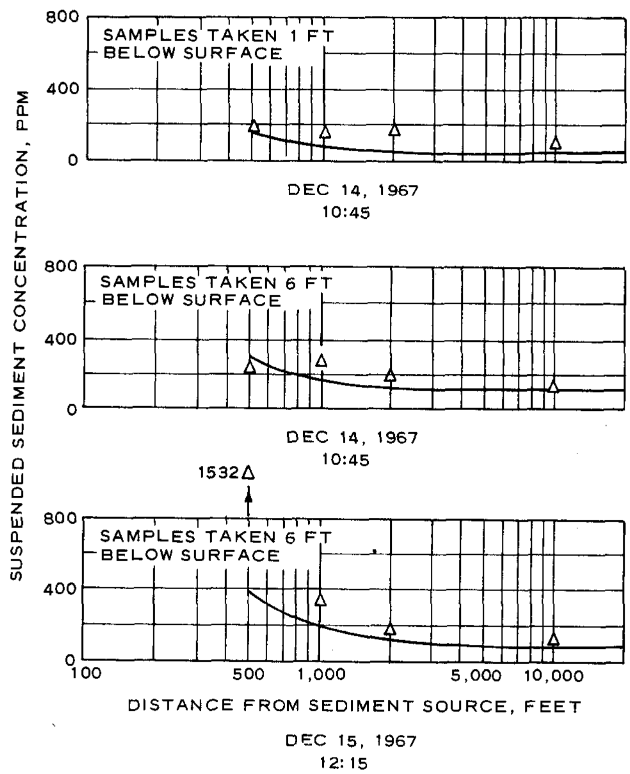

Fig. 3. Sediment concentration in the center of the sediment plume

distance of about 10,000 ft from the source both the computed and the measured results indicate essentially all material has settled.

Lawrence Livermore Hybrid Lagrangian-Eulerian Three-Dimensional Particle Diffusion Code: ADPIC

68. This model was developed for the calculation of the distribution of atmospheric pollutants under many different meteorological conditions, including transient stratified shear flow with complex surface boundaries.[35] The pollutant is represented statistically by many Lagrangian marker particles which are traced through an Eulerian grid. The particle trajectories are traced by vectorially adding the ambient velocity, the particle settling velocity, and a calculated diffusion velocity to account for turbulent diffusion. At each time step, the location of each particle is determined, and then linear interpolation yields the concentration at each mesh point of the Eulerian grid. Likewise, similar interpolation yields the velocity associated with each particle, which is then used to move the particle during the next time step. In its present form, ADPIC allows an Eulerian grid of up to 50 by 50 by 10 to be specified and is capable of handling up to 10^5 particles. For most previous studies, about 24 hours of real time was simulated in 15 minutes on the CDC 7600 for which the code is currently programmed. One of the chief advantages of using such a hybrid Lagrangian-Eulerian scheme is that it eliminates the artificial diffusion commonly encountered in purely Eulerian models.

69. Although the model was developed for atmospheric diffusion, it appears to be capable of application to aquatic environments with appropriate modification. Since the ambient velocity is allowed to be time and spatially dependent, possible application in an estuarine environment certainly appears feasible. It should be noted, however, that the model does not consider the initial dilution, as a result of initial buoyancy and momentum, of a cloud of material behaving initially as a unit. This initial dilution is of course dependent upon the method of introduction and can be very important in the study of the short-term fate of dredged material in the vicinity of the disposal site. Also, it

should be noted that ADPIC requires a three-dimensional ambient velocity field as input. There is some question as to how this would be obtained in an actual application.

Stanford Research Institute's Modeling Effort for Disposal in San Francisco Bay

70. The Stanford Research Institute has been awarded a contract by the U. S. Army Engineer District, San Francisco, to develop a model for the prediction of the fate of dredged material disposed of in San Francisco Bay. Development of the model has only recently been initiated, but from private communication with Mr. L. D. Spraggs[36] it appears that the basic idea behind the model will be the Lagrangian particle concept employed by Lawrence Livermore's ADPIC model. The ambient horizontal velocity components will be obtained from Leendertse's depth-averaged hydrodynamic model. These will then be used with Murray's[37] equation for the settling velocity of a particle (see Appendix A) to determine the particle trajectory. Mr. Spraggs has indicated that he intends to include a term in the settling velocity equation to account for vertical turbulence. He also indicated that perhaps some empirical expression for resuspension will be incorporated in the model. Present plans call for allowing the model to handle up to 1000 particles.

PART V: DISPERSION STUDIES RELEVANT TO RIVERINE DISPOSAL

71. Most mathematical models of transport phenomena in a river are one-dimensional water quality or sedimentation models. The dominant process in the transport equation is convection due to the cross-sectionally averaged ambient velocity. This then results in a longitudinal dispersion term in the governing equation representing the transport due to velocity shears, which as a result of the spatial averaging can no longer be handled in the convective transport term. As previously discussed, Fischer[14] has shown that in natural streams dispersion is due almost entirely to lateral variation in the downstream velocity.

72. As long as the pollutant fills the whole cross section of flow, the solution obtained from longitudinal dispersion equations should be compatible with that obtained from the three-dimensional time-averaged turbulent diffusion equation at such an asymptotic state, provided the determined longitudinal dispersion coefficient can accurately represent the real situation. Studies by Taylor,[12] Elder,[13] and Fischer,[14] as previously discussed, were concerned with determining expressions for the longitudinal dispersion coefficient. In the natural streams studied, Fischer[14] found that the dispersion coefficients varied from 50 to 700 rU^* where r is the hydraulic radius and U^* is the shear velocity. In a study of mixing characteristics of the Missouri River, Yotsukura et al.[38] found a longitudinal dispersion coefficient of approximately 16,000 ft^2/sec, which corresponds to about 5600 U^*d.

73. When using simple one-dimensional models to determine mass transport, it is essentially assumed that the method of introduction provides for immediate mechanical mixing over the stream cross section. Of course in practice, this assumption is never satisfied. Fischer found that, in terms of distance downstream from the injection of a tracer source, the one-dimensional dispersion model was a good approximation when

$$L > 1.8 \frac{\zeta^2}{r} \cdot \frac{\bar{U}}{U^*}$$

where ζ is the distance from the point of maximum surface velocity to the most distant bank, r is the hydraulic radius, and L is the distance downstream from the injection of the tracer source.

74. Again, as in the estuarine environment, no mathematical models that provide for detailed tracing of particulate matter beginning with its initial entry into the river have been found. However, Schroeder and his associates at Oregon State University[39] are currently involved in the development of such a model. It appears that a simplified approach that essentially treats particles as ballistic missiles is employed. The various discussions which follow pertain to mathematical studies of sedimentation in open channel flow. These are presented because at least parts should be relevant to the development of the dispersion phase of a model of the disposal of dredged material in rivers.

Chen's Formulation of the Longitudinal Dispersion Equation

75. A discussion of the derivation of a simplified form of the longitudinal dispersion equation was given in a previous section with no consideration given to the corresponding boundary conditions. Chen[40] has formulated the longitudinal dispersion equation for sediment in flow with a moving bed. Since the boundary conditions do involve a moving bed, some discussion of Chen's work seems appropriate.

76. Consider a system of sediment particles dispersing in an open channel flow with spatially and temporally varying boundaries as shown in fig. 4. The basic differential equation involving time-averaged concentrations and velocities, as previously given, is

$$\frac{\partial \bar{c}}{\partial t} + \bar{\vec{V}} \cdot \nabla \bar{c} = \nabla \cdot \left[(D + e) \nabla \bar{c} \right] \tag{12}$$

with corresponding time-averaged boundary conditions for sediment in flow given as, at $z = \zeta_1$, which is the expression for a variable streambed

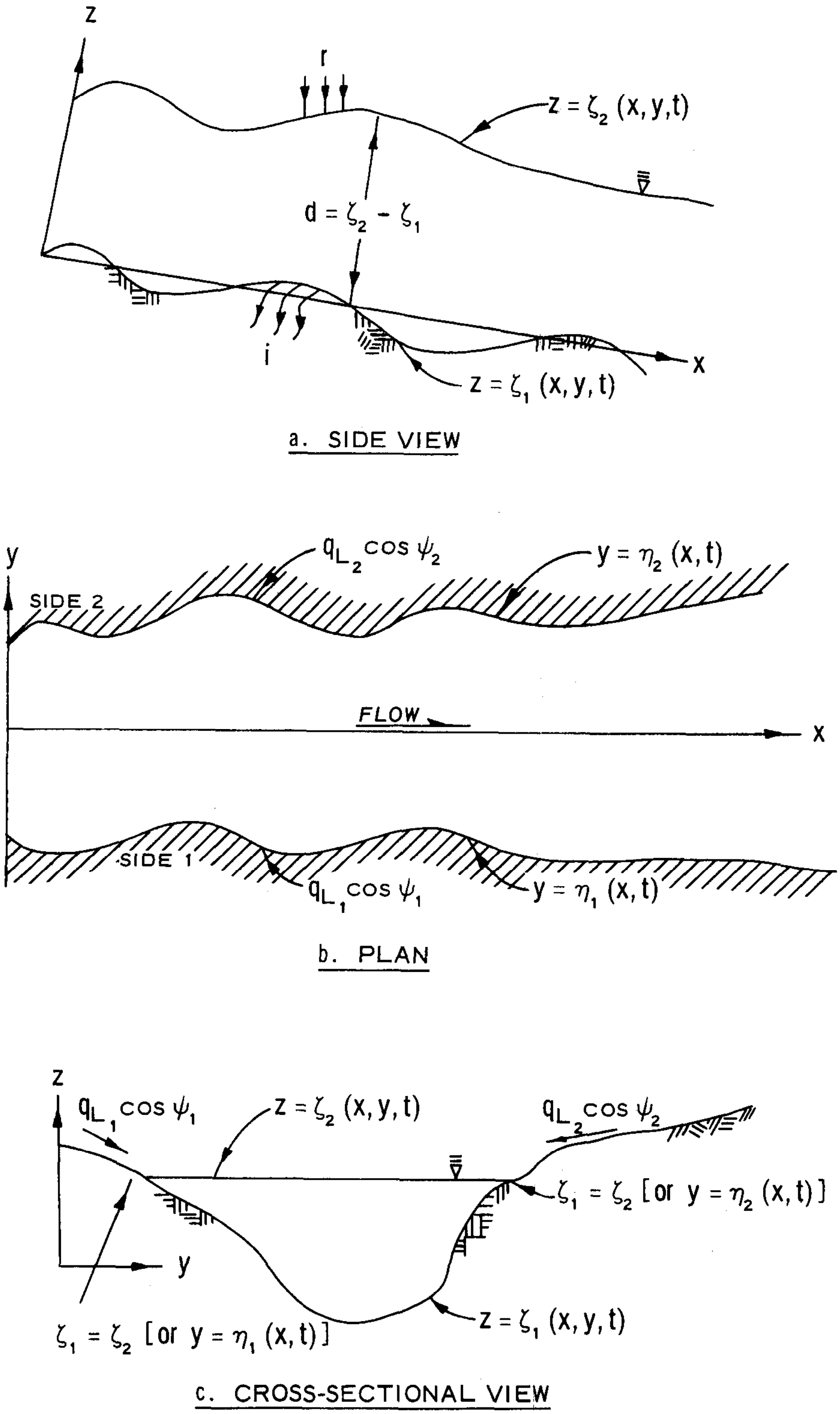

Fig. 4. Longitudinal dispersion of sediment in channel flow

$$\bar{c} = C_b \tag{13}$$

where C_b is the average concentration of sediment on the streambed, or

$$\bar{c} \frac{\partial \zeta_1}{\partial t} - \left[(D + e_1) \frac{\partial \bar{c}}{\partial x} \frac{\partial \zeta_1}{\partial x} + (D + e_2) \frac{\partial \bar{c}}{\partial y} \frac{\partial \zeta_1}{\partial y} - (D + e_3) \frac{\partial \bar{c}}{\partial z} \right] = -\bar{c}\overline{W}_s \tag{14}$$

and at $z = \zeta_2$, which is the expression for a variable water surface,

$$\bar{c} \frac{\partial \zeta_2}{\partial t} - \left[(D + e_1) \frac{\partial \bar{c}}{\partial x} \frac{\partial \zeta_2}{\partial x} + (D + e_2) \frac{\partial \bar{c}}{\partial y} \frac{\partial \zeta_2}{\partial y} - (D + e_3) \frac{\partial \bar{c}}{\partial z} \right] = -\bar{c}\overline{W}_s \tag{15}$$

and for flow of the water-sediment mixture, at $z = \zeta_1$,

$$\frac{\partial \zeta_1}{\partial t} + \bar{u} \frac{\partial \zeta_1}{\partial x} + \bar{v} \frac{\partial \zeta_1}{\partial y} - \bar{w} = i \tag{16}$$

and at $z = \zeta_2$,

$$\frac{\partial \zeta_2}{\partial t} + \bar{u} \frac{\partial \zeta_2}{\partial x} + \bar{v} \frac{\partial \zeta_2}{\partial y} - \bar{w} = f \tag{17}$$

where i is the infiltration rate and f is the rainfall rate . The first step in the derivation is to replace each of the dependent variables in the equations above with its average over the depth and some variation from that average. Equation 12 is then integrated over the depth of flow (i.e., $d = \zeta_2 - \zeta_1$) to obtain a two-dimensional dispersion equation with spatially and temporally varying boundary conditions.

77. Using depth-averaged values, boundary conditions are next formulated on both sides of the channel to include lateral inflows and outflows as, at $y = \eta_1$,

$$d \frac{\partial \eta_1}{\partial t} + \tilde{u}d \frac{d\eta_1}{dx} - \tilde{v}d = -\tilde{q}_{L_1} \cos \psi_1 \tag{18}$$

where η_1 is the expression for a variable left bank of a stream, $\tilde{q}_{L_1}$ is the lateral inflow, and ψ_1 is the angle between $\tilde{q}_{L_1}$ and the y axis,

$$\tilde{c}d \frac{\partial \eta_1}{\partial t} - E_x d \frac{\partial \tilde{c}}{\partial x} \frac{\partial \eta_1}{\partial x} + E_y d \frac{\partial \tilde{c}}{\partial y} = 0 \qquad (19)$$

where E_x and E_y are dispersion coefficients; and, at $y = \eta_2$,

$$d \frac{\partial \eta_2}{\partial t} + \tilde{u}d \frac{\partial \eta_2}{\partial x} - \tilde{v}d = \tilde{q}_{L_2} \cos \psi_2 \qquad (20)$$

$$\tilde{c}d \frac{\partial \eta_2}{\partial t} - E_x d \frac{\partial \tilde{c}}{\partial x} \frac{\partial \eta_2}{\partial x} + E_y d \frac{\partial \tilde{c}}{\partial y} = 0 \qquad (21)$$

where η_2 is the expression for a variable right bank of a stream, $\tilde{q}_{L_2}$ is lateral inflow, and ψ_2 is the angle between $\tilde{q}_{L_2}$ and the y axis. A tilde over a variable denotes a depth-average value. Now, assuming that the depth-averaged values in both the two-dimensional transport equation and boundary equations 18-21 can be replaced by a laterally averaged value plus some deviation, and then integrating the transport equation from $y = \eta_1$ to $y = \eta_2$, the one-dimensional longitudinal dispersion equation applicable to an open channel with a moving bed becomes

$$\frac{\partial C}{\partial t} + U \frac{\partial C}{\partial x} = \frac{1}{A} \frac{\partial}{\partial x} \left(EA \frac{\partial C}{\partial x} \right) + \Phi \qquad (22)$$

where either

$$\Phi = \frac{C_b}{A} \int_{\eta_1}^{\eta_2} \left(\overline{W}_s + \frac{\partial \zeta_1}{\partial t} \right) dy \qquad (23)$$

or

$$\Phi = \frac{1}{A} \int_{\eta_1}^{\eta_2} \left[\left(D + e_{cx} \right) \frac{\partial \overline{c}}{\partial x} \frac{\partial \zeta_1}{\partial x} + (D + e_{cy}) \frac{\partial \overline{c}}{\partial y} \frac{\partial \zeta_1}{\partial y} - (D + e_{cz}) \frac{\partial \overline{c}}{\partial z} \right] \Bigg|_{z=\zeta_1} dy \qquad (24)$$

(where e_{cx} , e_{cy} , and e_{cz} are sediment diffusion coefficients in the x , y , and z directions), depending upon whether equation 13 or 14, respectively, is employed at $z = \zeta_1$. Note that the effect of the moving bed is represented by a source term in the longitudinal dispersion equation.

78. A basic assumption in the development above is that particle fall velocities in turbulence are the same as those in a quiescent fluid. In addition, all particles are assumed to possess the same fall velocity. A discussion of particle settling velocities is presented in Appendix A.

Sayre's Study on the Dispersion of Silt Particles in Open Channel Flow

79. Sayre[41] has investigated the dispersion of silt particles in a uniform two-dimensional open channel flow. Various assumptions include:

- a. Turbulence does not affect the average fall velocity.
- b. The Von Karman-Prandtl velocity distribution, which is a log function of the depth, is employed for the ambient velocity.
- c. Turbulence is isotropic, i.e. $\varepsilon_x = \varepsilon_y = \varepsilon\psi(y)$, where ε_x and ε_y are turbulent diffusion coefficients.

With the above assumptions, the basic laterally averaged diffusion equation is nondimensionalized and written in a coordinate system moving downstream. In addition, boundary conditions formulated at the water surface and the channel bottom are similarly made nondimensional.

80. The nondimensional diffusion equation, together with the appropriate initial and boundary conditions, if solved, would give a complete picture of the dispersion process, not only for particles that remain in suspension, but also for particles that are stored on the bed and ones that are transported intermittently in suspension. However, due to the required computer time, Sayre elects to employ the Aris moment transformation previously discussed in connection with the Koh-Chang model. The dependent variables are consequently no longer a function of the dimensionless parameter for longitudinal position. Once again, the price for this reduction in the number of independent variables is that the solutions give the moments of the longitudinal distribution of dispersant rather than the distribution itself.

81. With a dimensionless time step of $\Delta\tau = 0.0005$ and a dimensionless depth step of $\Delta\eta = 0.05$ ($\eta = 1$ corresponds to depth of uniform flow), Sayre found that 13 seconds on a CDC 6600 and 1.3 minutes on an IBM 7094 were required for computations out to $\tau = 3$. In addition to mass transfer coefficients, the input consists of a value for the Von Karman turbulence coefficient, the dispersant fall velocity, and a bed absorbency factor and entrainment rate coefficient associated with the bed boundary equation.

Apmann and Rumer's Study of the Entrainment and Transportation of Sediments in an Open Channel

82. Apmann and Rumer[42] investigated the entrainment and transportation of mineral sediments from the bottom of an open channel in a two-dimensional flow field using the convective-diffusion equation as a mathematical model. The flow system was idealized so that the flow depths were considered independent of longitudinal distance and the flow was considered steady. With the additional assumption that the velocity and diffusivity are constant, the governing equation, after normalizing, becomes

$$\frac{\partial^2 \bar{c}'}{\partial y'^2} + Z\frac{\partial \bar{c}'}{\partial y'} = R\frac{\partial \bar{c}'}{\partial x'}$$

where

$$\bar{c}' = \frac{\bar{c}}{C_o} \; ; \; x' = \frac{x}{y_o} \; ; \; y' = \frac{y}{y_o} \; ; \; Z = \frac{W_s y_o}{\sqrt{\bar{\varepsilon}}} \; ; \; R = \bar{U} y_o / \sqrt{\bar{\varepsilon}}$$

and

C_o = characteristic concentration

y_o = the depth of flow

$\bar{\varepsilon}$ = mean value of vertical diffusivity

$\bar{U}$ = mean value of ambient velocity

83. A finite difference solution of the governing equation, satisfying the following boundary conditions:

a. At $x = 0$ and for all $y > 0$ ($y = 0$ at the bed); $\bar{c} = 0$, i.e. no sediment has yet been entrained;

b. At $y = 0$ and for all $x > 0$, $\bar{c} = C_o$; and

c. At $y = y_o$ and for all $x > 0$, the net transport through the surface is zero;

was obtained, although Mei[43] had earlier obtained an analytical solution.

Bonham-Carter and Sutherland's Three-Dimensional Model for Diffusion and Settling of Sediments at River Mouths

84. This model was developed mainly for the simulation of the buildup of deltas at river mouths.[44] Two basic assumptions are made in the structure of the model:

a. Diffusion processes at river mouths are similar to those found in jets.

b. The transport of suspended sediment can be treated statistically by considering the movement of nominal sediment particles.

Additional assumptions are that the river flow is steady and uniform and that a logarithmic velocity profile for the open channel turbulent flow can be employed. Neglecting lateral boundary effects due to shear at the river banks enables the assumption that the velocity profile remains the same at any point across the width of the river. Plane jet theory is then used to determine velocities in front of the river mouth. This is accomplished by treating the turbulent fresh-water layer flowing over the stationary salt water as a stack of thin horizontal plane jets. From this, the velocity can be determined for any point in the three-dimensional mass since the initial velocity at the mouth of each jet layer is known from the logarithmic profile.

85. The river is divided into a number of stream tubes with square cross sections, and the sediment content of each tube is represented statistically as a nominal particle traveling at the center of the tube. Depending upon their height from the channel bottom, these nominal particles represent certain proportions of the total sediment load. These fractions of the total load are given by the sediment

discharge profile, which is determined from equations presented by Vanoni et al.[45] for variations in velocity and concentration with depth in an open channel. Sediment concentration at a given elevation is assumed to remain constant along the channel, i.e., the nominal particle trajectories are straight lines. However, once beyond the mouth, trajectories begin to bend downwards, and, in addition, particles are dispersed in plan view at the same rate as the dispersion of momentum. The particle trajectories are calculated by assuming that their slope is the ratio of settling velocity to forward velocity. Having reached the salt water layer, particles are assumed to sink vertically.

86. The sediment deposits (sediment load may be divided into three grain-size fractions) can be allowed to build forward through several increments of time. The settling trajectories are automatically adjusted as the sediment floor builds up. Therefore, a dynamic response to changes through time is incorporated into the program. A similar three-dimensional model for the computation of sediment deposit and buildup at river mouths has been developed by Waldrop.[46]

Schroeder's Model for River Disposal of Dredged Material

87. Schroeder and his associates at Oregon State are currently about halfway through a 2-year study to develop a model for predicting the immediate fate of sandy soils discharged in water by pipeline dredging.[39] The underlying assumption in the model development is similar to that made by Bonham-Carter and Sutherland in that the slope of the trajectory of a sand particle in water is the vector sum of the ambient water, the particle velocity induced by the discharge, and the particle settling velocity. To investigate the effectiveness of the above assumption, an analysis of laboratory data using such a velocity model has been undertaken. Schroeder has indicated that calculated results from the model agree quite well with data obtained from laboratory experiments.

88. Recently, Schroeder, in cooperation with the U. S. Army

Engineer District, Portland, conducted a series of field experiments in the Columbia River to attempt confirmation of the model. Additional tests are planned in the near future. More detailed information about the model and the verification tests will be obtained as reports become available.

PART VI: CONCLUSIONS AND RECOMMENDATIONS FOR ADDITIONAL RESEARCH

89. This study has been an attempt to determine through a literature search of such technical journals as Water Resources, ASCE journals, The Journal of Fluid Mechanics, etc., if mathematical models for the determination of the physical fate of dredged material released in an aquatic environment exist. Personal contacts with research groups such as Tetra Tech, Inc., Environmental Sciences and Engineering, and EPA Offices proved helpful. A study of existing models plus an investigation of rather general mass transport studies which seemed relevant to mathematical modeling of dredged material followed. It was decided that perhaps the best way to organize the results from the investigation was by grouping the studies relevant to ocean, estuarine, and riverine disposals separately, since environmental conditions range from those in the ocean where turbulent diffusion is the major transport mechanism to estuaries in which tidal effects and density currents play a role.

90. By far the most significant dredged material disposal modeling work which has been performed to date is that of Koh and Chang while employed at Tetra Tech, Inc. Their efforts produced a model for the prediction of dispersion and settling in barged ocean disposal of wastes. The model is extremely well developed from a conceptual viewpoint and allows the dredged material to be either instantaneously dumped as a three-dimensional lump or pumped through a pipe underneath a moving barge. In either case, the material undergoes the possibility of three phases; namely, convective descent, dynamic collapse (including the possibility of bottom encounter), and long-term diffusion. As previously noted, the model's major limitation seems to stem from the employment of the Aris Moment transformation of the basic three-dimensional diffusion equation, which limits the applicability of the model to an ocean environment. In addition to the above limitation, no field verification or sensitivity analysis of the various parameters contained within the model is yet available. It is understood that the EPA is currently engaged in verification efforts.

91. No models capable of tracing the dredged material from its initial release into the water column until it settles to the bottom have been found for the estuarine or riverine environments. However, limited discussions of various dispersion studies in these environments were presented. These were considered to be relevant since any spatially averaged mathematical model of the disposal into such environments of material composed of small particulate matter such as clays or silt would not be complete without accounting for the dispersive transport. It may well be, however, that, since most of the material dredged and subsequently released into inland waterways consists of sand and gravel, a spatially averaged model that neglects the transport due to dispersion and thus considers only convection and settling of the dredged material would be applicable in the riverine environment. The model being developed by Schroeder at Oregon State University, which is expected to be applied to additional disposal operations in the Columbia River in the near future, appears to be designed along these lines.

92. As in the previous discussions, the following presentation of recommendations for future research on mathematical modeling of the disposal of dredged material can best be presented by discussing research needs for the ocean, estuarine, and riverine environments separately.

Research Needed in the Ocean Environment

93. No further model development work is recommended for the ocean disposal problem at this time. The major effort needed here lies in testing the Koh-Chang model. The Pacific Northwest Water Laboratory of the EPA is currently involved in planning some field verification work using disposal operations in the New York Bight. As previously mentioned, the Office of Dredged Material Research (ODMR) is going to select several disposal sites at which extensive monitoring studies will be conducted. Therefore, ODMR has an excellent opportunity to provide additional verification data at a minimum cost through some of the planned monitoring studies. As a result, it is recommended that the Koh-Chang model be obtained and made operational on the computer

facilities available to the U. S. Army Engineer Waterways Experiment Station (WES). Having the model operational at WES should be very helpful in the planning as well as operational phase of the monitoring studies at the selected ocean disposal sites. It might also be noted that having the model available for trial runs using preliminary data from prospective sites would be of great aid in the selection of some of the sites, using model applicability as a criterion. As a final note, no detailed sensitivity analysis of the model has been performed. It is believed that this is a vital part of the testing of any mathematical model, and it is therefore highly recommended.

Research Needed in the Estuarine Environment

94. In the estuarine environment, convection by the temporally and spatially varying ambient velocity field is the dominant mass transport process. Turbulent diffusion is not nearly as important in estuaries as in the oceans. However, if a spatial averaging over the velocity field is performed, then dispersion terms containing coefficients similar to diffusion coefficients may need to be considered in the basic transport equation.

95. In the Koh-Chang model, no spatial variation in the horizontal directions or time variation of the ambient velocity field is allowed. In order to apply the Koh-Chang model to disposal operations in an estuary, this spatial and time dependence must be incorporated. The model could probably be modified to allow the velocities to become spatially and temporally dependent in all phases of the bottom dump disposal operation without too much difficulty; however, in the jet convection phase of the jet discharge analysis, steadiness of the ambient velocity is an assumption that cannot be easily removed. It should be noted, however, that if the jet discharge is for only a short period of time, such as would be the case of a moving barge pumping dredged material through a pipe, steady conditions could probably be allowed. Then, allowance for the time and horizontal variation of the ambient current in the longer-term advective-transport phase could be made. In the case

of a continuous discharge through a fixed pipeline, a different approach should probably be taken. In either case, i.e. modification of the Koh-Chang model or development of a new approach, the problem is one of obtaining a spatially and temporally varying velocity field to be used as input. It appears that this dictates the use of a hydrodynamic model to provide the velocity field to be input into the transport model.

96. At this point, it should be noted that ODMR's major concern in the fate prediction of dredged material is with being able to make predictions over relatively short periods of time (2 to 3 days) in the vicinity of the disposal site rather than in the development of a general estuarine sediment transport model.

97. Based upon the above discussion, the following are presented as ideas for research that would greatly enhance the capability to predict the physical fate of dredged material disposed of in an estuarine environment:

- a. For the case of a bottom dump disposal or a short-term jet discharge from a moving barge, perhaps the Koh-Chang model should be modified to accept a spatial and temporal dependence of ambient velocities. This means dropping the Aris Method of Moments transformation and working with either the three-dimensional advective-transport equation or a model such as ADPIC based upon the Lagrangian particle concept.

- b. A model should be developed to handle the continuous pumping of dredged material over an extended period of time. If it can be assumed that the initial momentum and buoyancy of the dredged material are not significant factors in the final fate of the material, perhaps a multiphased model based upon the advective-transport equation, with the pipeline as a point source, would be sufficient. The development of such a model would be very similar to the modifications needed in the Koh-Chang model. If the initial discharge of the material is a significant factor in the transport, a model which takes into consideration the magnitude of the initial velocity of the dredged material and its orientation may be sufficient. This of course is currently allowed for in the Koh-Chang model for the jet discharge of dredged material over a short period of time. For the continuous release problem, perhaps an approach similar to that taken by Schroeder in the riverine environment or Lawrence Livermore's ADPIC code could be used.

c. Once the steps above are taken, laboratory and field verification studies must naturally follow. Also, a sensitivity analysis to determine the importance of coefficients which must be input should be performed.

98. It should be realized that a hydrodynamic model will have to be employed at the disposal sites to provide the velocity field to be input into the transport model. For example, a two-dimensional depth-averaged model such as Leendertse's[47] might be used to yield the horizontal velocities as a function of time and horizontal spatial coordinates. Transport due to velocity shear in the spatially averaged depth coordinate could be taken into account either through the use of a dispersion coefficient or perhaps by inputing the velocities as

$$U(x,y,z,t) = \tilde{U}(x,z,t)\ [1 + \varepsilon_u(y)]$$

$$W(x,y,z,t) = \tilde{W}(x,z,t)\ [1 + \varepsilon_w(y)]$$

where $\tilde{U}$ and $\tilde{W}$ would be given by the hydrodynamic model and the distribution functions ε_u and ε_w would be estimated from an inspection of recorded data. It should be realized that these functions are essentially taking the place of dispersion coefficients.

Research Needed in the Riverine Environment

99. Dredged material is disposed of in rivers by means of discharge from pipelines. As previously noted, convection by the ambient velocity is by far the dominant factor in the transport of mass in a riverine environment. Turbulent diffusion is of little importance in riverine transport phenomena. This is especially true in the case of dredged material since the dredge spoil released into rivers is normally composed of larger particulate matter such as sand and gravel. If the only interest is in a rough estimate of material concentration within specified longitudinal distances from the disposal point, a multiphase longitudinal dispersion model in which the pipeline is treated as a point source would probably suffice. However, if more detailed

information about the deposition of the dredged material is desired, perhaps a model such as that being developed by Schroeder, in which the trajectories of the particulate material are determined from a ballistic missile approach, should be considered.

100. At the present time it is believed that, rather than initiating new model development for the fate prediction of dredged material in the riverine environment, the model being developed by Schroeder should be thoroughly investigated upon completion of its development and verification for possible use by the Corps of Engineers.

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APPENDIX A: SETTLING VELOCITIES OF SEDIMENT PARTICLES IN A WATER COLUMN

1. The settling velocity of a particle directly characterizes its reaction to flow. Quite naturally, the size of the sediment grain influences its settling or fall velocity. Because the sizes of grains making up sediment vary widely, it has been found convenient to group sediments into different size classes or grades. Table A1 illustrates a grade scale proposed by the Subcommittee on Sediment Terminology of the American Geophysical Union.[48] Although sediment size is extremely important, the size alone is usually not sufficient to describe a particle. Other characteristics are shape and roundness. As defined by Wadell,[49] the shape is expressed in terms of sphericity, which is the ratio of the diameter of the circle with an area equal to that of the projection of the grain when it rests on its larger face to the diameter of the smallest circle circumscribing this projection. Roundness depends on the sharpness or radius of curvature of the edges. In studies by McNown and Malaika[50] and Albertson,[51] particle shape is expressed by a shape factor

$$SF = \frac{c}{\sqrt{ab}}$$

where a , b , and c are the lengths of the longest, intermediate, and shortest mutually perpendicular axes of the particle. It was found that the fall velocity could be expressed in terms of the shape factor, Reynolds number, and the nominal diameter, which is the diameter of a sphere of the same volume as that of the given particle.

2. Graf and Acaroglu[52] have investigated the settling velocities of natural grains. A brief review of several theoretical studies of the fall velocity of spheres at very small Reynolds numbers ($R_e < 2$) is given along with several empirical equations which give good results at higher Reynolds numbers. In addition, a critical discussion of Rubey's equation

$$W_s = \frac{\sqrt{9\mu^2 + \frac{4}{3} R^3(\rho_p - \rho)g} - 3\mu}{R\rho}$$

where

W_s = settling velocity

μ = dynamic viscosity

R = particle radius

ρ_p = particle density

ρ = fluid density

g = gravitational acceleration

is also presented. Fig. A1 consists of plots of settling velocity W_s versus particle diameter j as determined from theoretical equations by Stokes and Newton and Rubey's equation and from data for different sphericities as given by Pettyjohn and Christiansen.[53] A plot obtained from data by Mamak[54] using natural sand with a density of 2.65 settling in water ($\rho = 1.0$) at a temperature of 20°C is also presented in fig. A1. Note that, for particles with a sphericity of $\psi = 0.670$, Rubey's

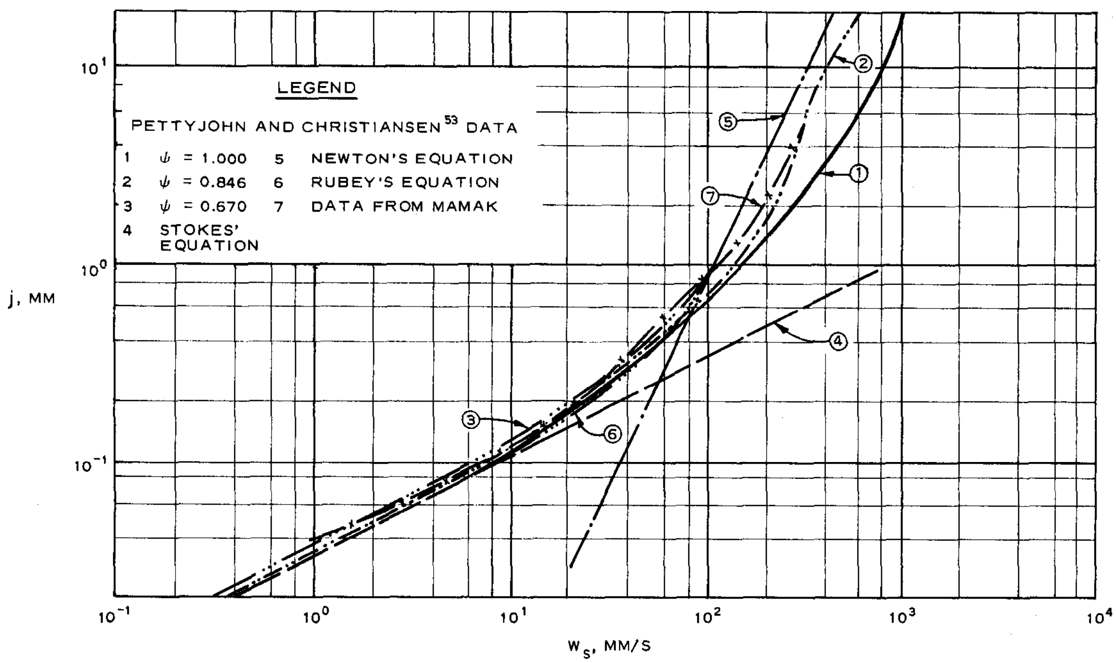

Fig. A1. Settling velocity versus diameter with shape factor ψ for quartz ($\rho = 2.65$) at 20°C

equation gives good results. Graf and Acaroglu conclude that for odd-shaped sediment grains, table A2 (after Mamak[54]) should be used to determine settling velocities, whereas, should the sphericity of the particles be known, the experimental curves of Pettyjohn and Christiansen[53] should be employed. It should be emphasized that these results are only for natural sand sediment.

3. Watson[55] has attempted to modify Rubey's equation with the inclusion of two empirical constants. This modified form of Rubey's equation becomes

$$W_s = \frac{\sqrt{9Z^2\mu^2 + \frac{4}{3} XR^3(\rho_p - \rho)g} - 3Z\mu}{XR\rho}$$

where $Z = 0.622$ and $X = 0.5305$ were determined from experimental data. Watson concludes that the fall velocity for groups of sediment grains can be computed for any fluid, provided the particle density and radius and the fluid density and viscosity are known.

4. When there are a number of particles dispersed in a fluid, the fall velocity will differ from that of a single particle due to the mutual interference of the particles, i.e., hindered settling. In a study by McNown and Lin,[56] it was determined that, for the case of uniform quartz spheres and sand settling in water with even moderate concentrations (1 or 2 percent by weight), the correction in the fall velocity becomes significant. One of the faults Acaroglu[57] finds with Watson's modified Rubey equation is that he doubts if the effect above is included. However, as Watson[58] notes, the modified Rubey equation was determined using data for groups of sediment grains and thus does to a certain extent include the hindered settling effect.

5. In some suspensions of clay and silt, electrochemical forces tend to hold particles together once they come into contact. Once two or more particles combine, they will settle as a group with higher velocity than any of the individual particles of the group falling alone. If turbulence is present, the particles can be brought into contact by the mixing that always occurs in such an environment. However, the agitation due to turbulence may also act to tear apart agglomerations of particles brought together initially by the turbulence. Thus, the average fall velocity of material subject to flocculation would be expected to initially

increase and then to decrease as the turbulence acts to break the flocs up. This behavior is illustrated in fig. A2 taken from McLaughlin.[59]

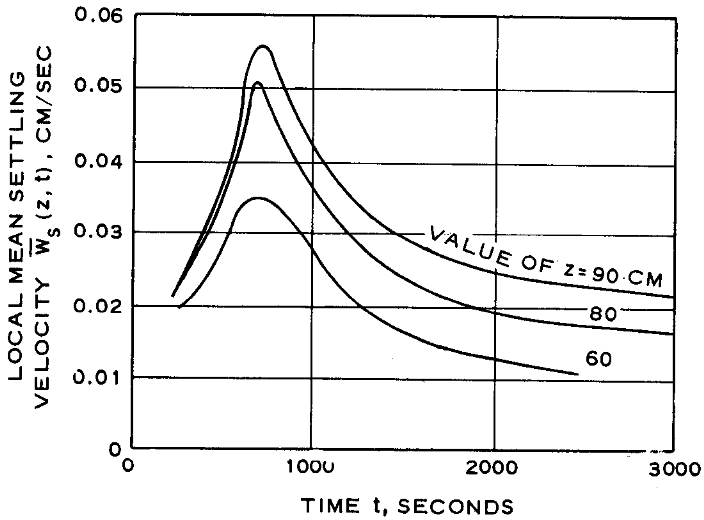

Fig. A2. Local mean fall velocity as a function of time at depths for bentonite clay and alum in water

6. Murray[37] has studied, both theoretically and experimentally, the effects of turbulence on the settling velocity of sediment grains. A simple theoretical model for the settling of particles was constructed to be

$$\frac{dW_o}{dt} = - \frac{3C_d\rho|W_o|W_o}{4j\rho_p} - g\frac{\rho_p - \rho}{\rho_p} + \frac{dW}{dt}\left(\frac{\rho}{\rho_p} - 1\right) - k\frac{\rho}{\rho_p}\frac{dW_o}{dt}$$

where

W_o = $(W_p - W)$, the relative particle speed, where W_p is the instantaneous particle velocity

C_d = drag coefficient

ρ = fluid density

j = particle diameter

ρ_p = particle density

g = gravitational constant

W = vertical water velocity

k = coefficient of added mass

The equation above was used to investigate the behavior of quartz sand particles under natural flow conditions by letting $W = Z_o \omega \sin \omega t$ simulate the turbulence, where Z_o is the amplitude and ω is the angular frequency. Computed results indicated that oscillating water greatly reduces particle fall velocity and that the effect is magnified with increasing frequency of oscillation. The experiments reported by Murray verified the theoretical results by showing the settling velocity reduction to be greatly affected by the high-frequency components of the grid-produced turbulence. The average velocity of fall determined by experiments in various turbulent fields was reduced by as much as 30 percent below the corresponding still-water terminal fall velocity.

Table A1

Sediment Grade Scale (After Reference 48)

	Size Range				Approximate Sieve Mesh Openings per Inch	
Class Name	Inches	Millimeters	Microns	Inches	Tyler	United States Standard
Very large boulders		4096 to 2048		160 to 80		
Large boulders		2048 to 1024		80 to 40		
Medium boulders		1024 to 512		40 to 20		
Small boulders		512 to 256		20 to 10		
Large cobbles		256 to 128		10 to 5		
Small cobbles		128 to 64		5 to 2.5		
Very coarse gravel		64 to 32		2.5 to 1.3		
Coarse gravel		32 to 16		1.3 to 0.6		
Medium gravel		16 to 8		0.6 to 0.3	2-1/2	
Fine gravel		8 to 4		0.3 to 0.16	5	5
Very fine gravel		4 to 2		0.16 to 0.08	9	10
Very coarse sand	2 to 1	2.000 to 1.000	2000 to 1000		16	18
Coarse sand	1 to 1/2	1.000 to 0.500	1000 to 500		32	35
Medium sand	1/2 to 1/4	0.500 to 0.250	500 to 250		60	60
Fine sand	1/4 to 1/8	0.250 to 0.125	250 to 125		115	120
Very fine sand	1/8 to 1/16	0.125 to 0.062	125 to 62		250	230
Coarse silt	1/16 to 1/32	0.062 to 0.031	62 to 31			
Medium silt	1/32 to 1/64	0.031 to 0.016	31 to 16			
Fine silt	1/64 to 1/128	0.016 to 0.008	16 to 8			
Very fine silt	1/128 to 1/256	0.008 to 0.004	8 to 4			
Coarse clay	1/256 to 1/512	0.004 to 0.0020	4 to 2			
Medium clay	1/512 to 1/1024	0.0020 to 0.0010	2 to 1			
Fine clay	1/1024 to 1/2048	0.0010 to 0.0005	1 to 0.5			
Very fine clay	1/2048 to 1/4096	0.0005 to 0.00024	0.5 to 0.24			

Table A2

Rate of Drop Down of Sediment Grains (After Mamak[54])

Laminar Motion		Transient Motion	
Grain Diameter, mm	Settlement Velocity, mm/s	Grain Diameter, mm	Settlement Velocity, mm/s
0.010	0.066	0.15	14.82
0.015	0.149	0.20	20.42
0.020	0.265	0.30	31.62
0.030	0.597	0.40	42.92
0.04	1.06	0.5	54.02
0.05	1.66	0.6	65.22
0.06	2.39	0.7	76.42
0.07	3.25	0.8	87.62
0.08	4.24	0.9	99.02
0.09	5.87	1.0	110.02
0.10	6.63	1.2	132.42
0.12	9.56	1.5	166.02
0.15	14.90		

Turbulent Motion			
Grain Diameter, mm	Settlement Velocity mm/s	Grain Diameter, mm	Settlement Velocity mm/s
1.50	164.4	9.0	403
1.75	178.0	10.0	425
2.0	190.0	12.5	477
2.5	212.5	15.0	520
3.0	232.5	17.5	562
4.0	268.5	20.0	602
5.0	300.0	22.5	637
6.0	329.0	25.0	672
7.0	355.0	27.5	706
8.0	380.0	30.0	736

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13. ABSTRACT

A literature search of technical journals coupled with contacts with other research groups has revealed that very little mathematical modeling of the physical fate of dredged material disposed of in an aquatic environment has been undertaken. The most significant modeling effort that has been found is a mathematical model for prediction of dispersion and settling in barged ocean disposal of wastes developed by R. C. Y. Koh and Y. C. Chang. This model allows for disposal of dredged material by instantaneous bottom dump as well as pumping the material through a pipe under a moving barge. In both disposal operations, the material is traced through three possible phases; namely, convective descent, dynamic collapse, and long-term diffusion. The dynamic collapse is also generalized to account for the possibility that the cloud hits the bottom. The major limitations of the model appear to be: (a) The model was strictly developed to study disposal in an ocean environment; and (b) There has been only limited laboratory and no field verification of the model; however, it should be noted that the model is conceptually well designed. For estuarine and riverine environments, no models capable of tracing dredged material from its initial release into the water column until it is stored on the bottom have been found. However, for the riverine environment, Schroeder and his associates at Oregon State University are currently developing a mathematical model for tracing dredged material released by pipeline discharge. The model is based upon pipeline discharge velocity, ambient fluid velocity, and particle settling velocity. Additional information should be obtained concerning the development and verification of this model to assess its applicability. As a result of the investigation of identified models and relevant transport studies, the following recommendations are offered: (a) In the ocean environment, sensitivity analyses and field verification of the Koh-Chang model are needed; (b) Model development in the area of predicting the short-term fate of dredged material in the vicinity of the disposal site is needed for the estuarine environment; and (c) No model development is recommended for the river disposal problem until further investigation of Schroeder's work is completed.

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